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Demonstrating Harmonics and Beats*

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A group of ten fixed-frequency audio oscillators having frequencies related by $110(N)$, where N is an integer, are used. Details of construction of oscillators, audio amplifier, and associated power supply are given. With the oscillators it is possible to demonstrate production of synthetic first and second harmonics by playing two or more tones simultaneously. Five characteristic tones may be played simultaneously to simulate a bell-like tone.

TEN fixed-frequency audio oscillators make a very handy way of showing many of the relations between musical tones. The associated power supply, audio amplifier, loudspeaker, and cathode-ray oscilloscope complete the equipment required (see Fig. 1).

A typical oscillator circuit[1] is drawn in Fig. 2, and a photograph of the ten units is reproduced in Fig. 3. The frequencies are $110(N)$, where $N=1, 2, 3\cdots10$. These were chosen so that the fourth harmonic, 440 cps, may readily be checked with a standard "*A*" pitch pipe.

Each oscillator has one of its capacitors "*C*" screw-driver-tuned. Thus each frequency can be changed slightly, either to retune after moving, or to give nonharmonic tones close to the primary ones.

DEMONSTRATIONS

1. Those who have an ear for music will recognize most of the ten harmonics—especially if the 7th and 9th are omitted. If the 7th appears to be nonharmonic (it *does* sound harsh when played with the others) 880, 770, 660, 550, and 440 should be played in that order, leaving them all turned on. At low volume this combination is acceptable as being musical. If played loudly, a different result follows.

2. Any tone, played loudly with its adjacent harmonic, produces a synthetic first harmonic (fundamental) tone. For example, when $(N)(110)$

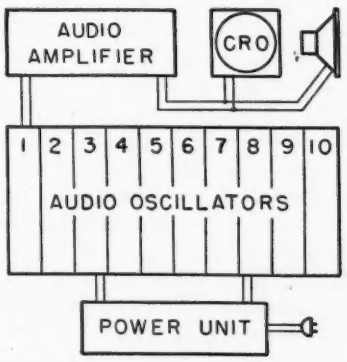

FIG. 1. Complete set-up block diagram.

* This article is based upon a paper delivered at the Twentieth Annual Meeting of the American Association of Physics Teachers, Barnard College, New York, February, 1951. See R. C. Hitchcock, Am. J. Phys. **19**, 329 (1951).

[1] Ginzton and Hollingsworth, Proc. Inst. Radio Engrs. **29**, 43–49 (1941).

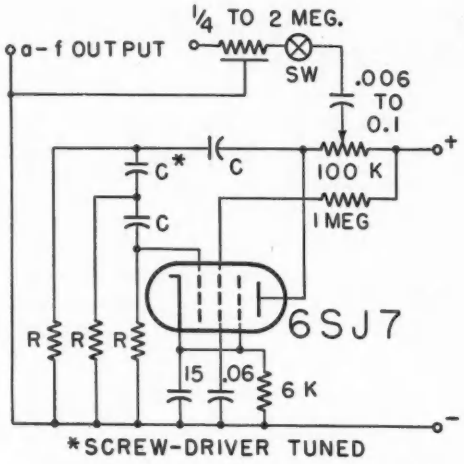

FIG. 2. Feedback oscillator, schematic circuit showing values.

and $(N+1)(110)$ are sounded, the difference of 110 cps is heard.

3. If $(N)(110)$ and $(N+2)(110)$ are sounded together, the difference tone of 220 cps is clearly audible at high volume.

4. Now for a more striking demonstration; sound 660 and 880, giving the 220 cps, *then add a higher harmonic*, 990. The 220-cps tone apparently goes an octave lower, to 110 cps, even though a *higher* harmonic is added.

5. As an example of nonharmonic tones, the first five characteristic tones of a bell can be played if one oscillator is retuned. Professor A. T. Jones[2] shows five of the bell tones to be 1/1.2/1.5/2.0/2.5; multiplying each by 4 we get 4/4.8/6/8/10. These are all harmonic except the 4.8, and by tuning the 5.0 down to 4.8 a bell-like tone is produced. For the musically inclined, an easy way is to tune down the 5.0 by a half-tone. Since each half-tone of the twelve note scale is the twelfth root of two (1.06) with respect to its adjacent half-tone—this gives 5/1.06=4.72, an approximation of the desired result.

A 100-$\mu\mu$f variable air capacitor, added since Fig. 3 was made, allows this changing of tone 5, by adding capacitance.

In Dr. Harvey Fletcher's article,[3] the first seven characteristic bell tones are 3/7/11/17/19/24/29; multiplying each by 0.1 (omitting the first two) produces figures very similar to those of Professor Jones. These are 1.1 with 1.2; 1.7 with 1.5; 1.9 with 2.0; 2.4 with 2.5; and 2.9 with 3.0: bell tones do not always have the same tonal content, and often their tones are not harmonic.

[2] A. T. Jones, Phys. Rev. **16**, 247 (1920).
[3] Harvey Fletcher, Am. J. Phys. **14**, 215–225 (1946).

OSCILLATOR DETAILS

Each oscillator uses a 6SJ7 sharp cut-off pentode, with circuit values shown in Fig. 2. The ten 6.3-v heaters are in parallel. To use ten 12.6-v heaters (12SJ7) in series on 126 v might seem desirable, but the published "heater to cathode" maximum of 90 v precludes their use on a 120-v line with its peak of 170 v.

For the oscillator[1] of Fig. 2:

$$f=1/(2\pi(6)^{\frac{1}{2}}RC)=0.065/RC,$$

where

f is cycles/sec, R is ohms, and C is farads.

For a frequency of 1000 cps, assuming $R=250{,}000$ ohms,

$$C=0.065/(250{,}000\times1000)=0.00026\mu\text{f}.$$

As shown in Fig. 2, three resistors R, and three capacitors C, comprise the phase-shift network in each oscillator unit. Commercial tolerances (plus or minus 20 percent) are satisfactory for operation, though not for frequency. That is, the oscillator will work, but exact frequency values require juggling R, or C, or both. In the oscillator units shown in Fig. 1, the middle condenser is screw-driver-tuned, a "trimmer" capacitor.

POWER UNIT DETAILS

The rectifier unit is in the general class of chain rectifiers of Cockroft-Walton,[4] and Greinacher, Bowers, and Cockroft.[5] The big improvement

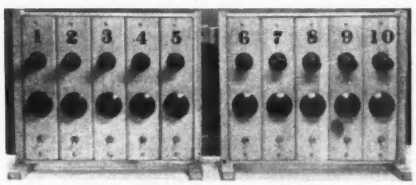

FIG. 3. Picture of ten-tone oscillator.

[4] E. W. Titterton, Wireless World **44**, 40 (1948), Fig. 3.
[5] A. E. Knowlton, "Chain rectifier circuit," *Standard Handbook of Electrical Engineering* (McGraw-Hill Book Company, Inc., New York, 1941), 7th ed., p. 2264.

over some "transformerless power supplies," is that the ac line input and the dc output have a common lead.[6,7] This prevents hum pickup in the negative dc lead, to which the sensitive oscillator tube grid is connected.

For safety a 60-watt, 120-v bulb is put in series with the negative lead. If by chance the audio amplifier is grounded, and the power supply connected incorrectly, this bulb lights. In this case, simply turn the power supply ac plug 180°. At the low drain of the 10-tone oscillator, the resistance of this bulb is only a few dozen ohms.

The 117Z6 rectifier comprises two separate diodes in one envelope, and has a heater which takes line voltage. At no-load this half-wave doubler gives about 320 v, and with the 10-ma drain of the ten tone oscillator, the potential difference is 180 v.

[6] R. C. Hitchcock, "Thermionic rectifier circuits," Electronics, **17**, 105 (1944), Fig. 5.
[7] O. T. McIlvaine, "Rectifier tube," U. S. Patent 1,946,354, February, 1934.

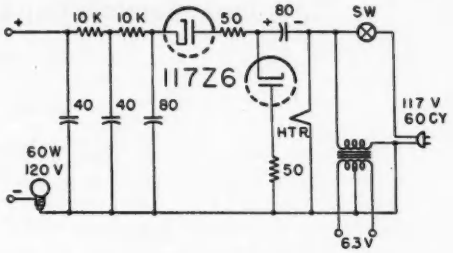

FIG. 4. Power supply, schematic diagram, giving values.

With the high value electrolytic capacitors used, 80 and 40μf (300-v dc rating) it is good practice to have a 50 ohm, ½-watt resistor in series with each plate of the 117Z6 to limit the initial charging current. Note the polarity of the first capacitor: negative goes to the ac line through the switch.

The heater transformer, 117 to 6.3 v, 3.0 amp, is conventional; it is the only heavy component in the power unit.

In this book it is my aim to try and clear myself of the imputation of esotericism by describing, as far as I may, the ideas at present current about the basic constituents of the physical world. There have been several excellent books recently written on the subject, but it has seemed to me that some of them, perhaps through attempting to cover too wide a field, do not always succeed in explaining their subject. They excite the wonder of the reader, by suggesting to him what extraordinary difficulties there are in the ideas of physics; they are like a conjurer whose tricks seem to us inexplicable. I have set myself what is, I think, a more ambitious task, for I want to try and show how the tricks are done. I shall count myself as having succeeded, if at the end of the book any surviving reader will speak no longer of the mysteries of science, but shall we say of the naturalness of Nature.
—C. G. DARWIN, *The New Conceptions of Matter*, 1931.

C. V. Boys' Rainbow Cup and Experiments with Thin Films

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(Received February 7, 1951)

The paper describes experiments on thin films that can be done easily with Boys' rainbow cup, mounted on a phonic wheel to insure constancy of rotational speed. Although the air within the cup, on one side of the film, has free access to the atmosphere on the other side of the film, the film itself is spherical. A quantitative explanation of the effect is given in terms of Bernoulli's principle.

IN 1912 C. V. Boys[1] demonstrated his little invention of the rainbow cup to the Royal Society of London.[2] It may not be known on this side of the Atlantic and so I wish to describe it and some of the experiments that may be made with it. The cup may be obtained from Griffin and Tatlock, Kemble St., Kingsway, London, W.1. The following is an excerpt from Boys' remarks: "The rainbow cup (Fig. 1) makes it possible to produce and observe effects more brilliant and varied than any that have hitherto been possible, even with the soap bubble. It has the advantage, also, that no special skill is required, so that its use need not be confined to accomplished experimentalists. It consists of a circular hollow brass box about 12.5 cm in diameter and 5 cm deep mounted on a hollow tube which encloses and rests on the top of a steel spindle fixed to the triangular base. The box can be set in rotation by drawing the finger over the knurled surface of the tube and the spin will remain for a long time."

FIG. 1. C. V. Boys' rainbow cup in rotation, showing film with (colored) rings.

The upper edge of the box is inturned giving a horizontal flange about 1.0 cm wide. It is pierced with 12 equidistant holes each about 5 mm in diameter lying in a circle of 11.5 cm radius. A transparent conical celluloid cover fits over the cup; it serves to keep off draughts and floating dirt. A soap film is stretched by the use of a celluloid wiper[3] over the inturned edge of the cup. With the cup in the hand and the film vertical the brilliant horizontal bands of color gradually appear as the film drains. These are fully described in the little book by Boys on *Soap Bubbles*[4] and illustrated by a colored plate.

In order to bring our the full brilliance of the colors the film should be observed in the light of a bright cloudy sky or of a brightly illuminated opal glass shade.

Boys continues: "Starting with the cup on the spindle and putting on a new film, put on the cover and spin gently. Do not be in too great a hurry and spin it as fast as possible. Gradually a colored ring pattern of alternate pink and green rings will be developed, each new color appearing at the center and expanding gradually. If the speed of rotation is reduced, the rings will collapse to a certain extent. On spinning faster again they will expand and new and brighter colors will appear at the center. The colors depend simply upon the thickness of the film[5] and

[1] Of soap bubbles and gravitation constant fame.
[2] C. V. Boys, Proc. Roy. Soc. (London) **87**, 340 (1912).
[3] The wiper is 12 cm long and about 4 cm wide. The soap solution is placed in a shallow long narrow trough about 13 cm long and 1 cm square cross section.
[4] C. V. Boys *Soap Bubbles, Their Colours and the Forces that Mould Them* (Macmillan Company, New York, 1924), p. 149.
[5] From the colored diagram in Boys' book the thickness of a film can be read by inspection.

the angle at which it is viewed. As the film becomes thinner the colors appear in the following order beginning with a film fifty millionths of an inch (125×10^{-6} cm) thick: pale green, pink, pale green, pink, bluish green, salmon color, bright green, magenta, yellowish and then brilliant apple green (20×10^{-6} inch), blue, purple, red, yellow, poor white, steel blue, purple, brown, straw-color, white, black, ($\frac{1}{2}\times10^{-6}$ inch)."

Boys' account proceeds to describe further experiments on the rings and spirals seen with the rotating film, on the black spots, on the effect of ammonia gas on the film, and on the effect of putting rings of hair and wetted birds' eggs upon the film; a fairly full account can be read in the pamphlet supplied by the maker of the apparatus. As I am partly color-blind the rings do not have all the colors described by Boys; they are predominantly either yellow or blue and rarely, if ever, have I seen a black film.

The portion of Boys' account in which I became interested reads somewhat as follows: "If when the small holes in the rim are open and the cover is not in place the box is turned gently the plane film will be drawn in more or less according to the speed of rotation" (Fig. 2b). The defect of pressure is due to some of the air escaping through the holes in the flange. This is often said to be due to centrifugal force. If the small holes are closed by films of their own, the film remains nearly plane on gentle rotation for the pressure of the air in the box remains practically atmospheric. Putting the cover in place, its wetted edge makes a seal and whether the small holes are open or not equilibrium of pressure below and above the film soon becomes established and the film remains plane. We say the film remains nearly plane when the holes in the ring are closed but if the cup is rapidly rotated it will be observed that the center of the film is depressed and a ring portion near the edge is elevated (Fig. 2a).

When the cover is off and the holes are open it is evident that on rotation some of the air in the box escapes through the holes thus leaving a defect of pressure within the box. How does the shape of the film depend upon the speed of rotation?

In order to get definite and steady speeds of rotation I used a Rayleigh synchronous motor

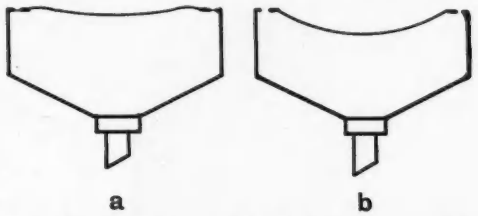
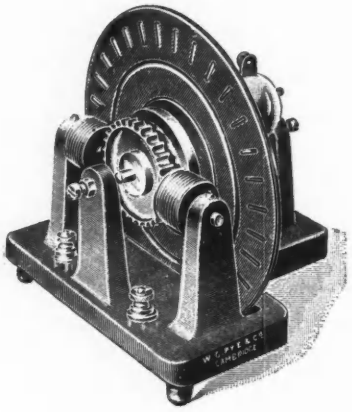


FIG. 2. Section of rotating rainbow cup and film, (a) with ring of small holes closed (b) with ring of small holes open.

or phonic wheel as made by W. G. Pye and Company of Cambridge, England. Two forms are shown in Fig. 3a and b. Of Fig. 3a the maker says, "This instrument consists of two fixed electromagnets and a rotating armature in the form of a soft iron wheel with 30 teeth. The spindle carrying the armature revolves on hardened steel centers and is fitted with a friction wheel which effectually prevents wandering and ensures perfectly smooth running. A timing device is fitted which rings a bell at every 100 revolutions of the motor spindle. The stroboscopic disk is of stout aluminum and has 30 slots. The instrument may be used to measure the frequency of a tuning fork or the frequency of the alternate current supply, and with either arrangement provides a steady rotation." The *modus operandi* is to rotate the wheel by pushing the finger across the knurled spindle until at the same time the right speed is reached and the cogs are in the correct position with respect to the magnet for the current coming from the intermittent fork-contact, or for the maximum of the ac supply, to receive an onward force sufficient to overcome the friction of the machine. When these occur together the finger may be withdrawn; the rotation is maintained. A fuller description is given in the pamphlet supplied by the maker with the apparatus.

Alternating current supplies of frequencies 60 and 25 cycles per second were available to me. With the 60-cycle, two speeds of the phonic wheel were obtainable, the bell ringing in 25.6 and 51.2 sec, respectively. With the 25-cycle only one speed was used, the bell ringing every 60.0 sec. To drive the cup, the form of the wheel shown in Fig. 3(b) was used; the little cross piece at the top was removed and a rod screwed on in its place. This rod carried the rainbow cup.



a

b

FIG. 3. The Rayleigh synchronous motor or phonic wheel: (a) form with axis horizontal, (b) form when axis may be horizontal or vertical.

The film being wiped in position all the little holes were cleared with a match stick and the phonic wheel set in rotation. To get the right speed one has to look through the slots of the wheel at one of the carbon filament lamps in the ac circuit. With a bar magnet in position near the lamp the filament (if sufficiently flexible) vibrates with the frequency of the current; the wheel must be speeded up until the filament appears stationary either in one position at the lower speed or in two positions at the higher speed. After some time the film takes a definite position. The radius of the spherical shape of the film was measured optically by two methods: (1) A little lamp in a tiny box was adjusted so that the image of the lamp fell back on the box. (2) A parallax method was used with a point-source of light. In both cases when the adjustment was as nearly perfect as possible the distance from the source to the rim of the cup was measured. The fact that an image was obtained showed that the film was spherical; poor reflecting power and a slight wobble made the readings approximate only. The values obtained for the radii were, with the 60-cycle higher speed, $9\frac{1}{2}$–10 cm; with the 60-cycle lower speed, 37–39 cm; and with the 25-cycle, 50–52 cm. Table I gives in the first column n, the frequency of rotation, in the second column the value of R, the radius of the surface of the film and in the third column the calculated values of n^2R. We see that n^2R is practically a constant, i.e., the radius of the film-surface varies inversely as the square of the speed of rotation. This result was not sought by Boys.

The solution used in the above experiment was not an ordinary soap solution. Professor Kuehner[6] gives a recipe for a solution which gives a more stable film and in these experiments repeated breakages of the film are apt to be a trifle annoying. Kuehner's solution is made up of cane sugar, glycerol, and aerosol. The surface tension of this solution was measured by Searle's simple torsion balance.[7] This little instrument, as made by Pye and Company of Cambridge, is shown in Fig. 4. Its principle of operation is obvious and it is a more convenient apparatus for soap solutions and other liquids which give fairly permanent films than the more complicated Du Noüy's apparatus. The surface tension of

TABLE I. The rainbow cup. Speeds of rotation and radii of curvature of the film.

n (sec^{-1})	R (cm)	n^2R
100/25.6 = 3.91	95	145
100/51.2 = 1.95	38	145
100/60 = 1.67	51	142

[6] A. L. Kuehner, J. Chem. Educ. **25**, 211 (1948).
[7] G. F. C. Searle, Proc. Cambridge Phil. Soc. **17**, 285 (1913); also Searle, *Experimental Physics* (Cambridge University Press, London, 1934), p. 171.

Kuehner's solution was the same as that of Boys' soap solution, namely about $26\frac{1}{2}$–27 dynes/cm.

With the higher speed of the phonic wheel on the 60-cycle ac, since $R=9.5$ cm the pressure in the box is less than atmospheric pressure by $4\times26\frac{1}{2}/9.5=11.2$ dyne/cm². Could this defect of pressure be calculated? How can one calculate the pressure within the rotating cup, closed except for the holes around the circumference? Also, how does the air pressure vary at different points within the cup?

One might make the attempt by applying Bernoulli's principle to a stream tube within the box and just underneath the holes and a stream tube just outside the box and above the holes. They may be considered to have the same energy. The tube within rotates with the speed of the holes, the tube above, in free air, will have only a small velocity. Applying the usual Bernoulli equation with common notation, we have

$$\tfrac{1}{2}\rho v_A^2+p_A+\rho g h_A=\tfrac{1}{2}\rho v_B^2+p_B+\rho g h_B,$$

where A, B, refer to the stream tubes below and above, respectively. Now v_B is nearly zero; also $h_A=h_B$. Therefore

$$\tfrac{1}{2}\rho v_A^2=p_B-p_A.$$

The radius of the circle passing through the centers of the holes is 5.75 cm. Therefore, at the higher speed on the 60-cycle ac,

$$v=2\pi\times(100/25.6)\times5.75$$

$$=141 \text{ cm/sec.}$$

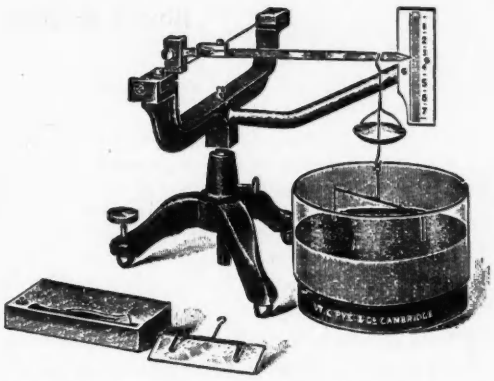


FIG. 4. Searle's simple torsion balances used in the measurement of the surface tension of a soap solution.

Taking $\rho=0.00118$ g/cm³,

$$p_B-p_A=\tfrac{1}{2}\times1.18\times10^{-3}\times141^2=11.7 \text{ dyne/cm}^2.$$

The agreement is good. Thus it appears that this is a feasible method of calculating the pressure of the air in the stream tube A. The approximate spherical shape of the film indicates that the defect of pressure is nearly the same under all parts of the film. This spherical film dips down 1.6 cm into the box and it is interesting to note that the pressure of a column of air 1.6 cm high is equal to 2.4 dynes/cm².

With the spindles of the phonic wheel and cup horizontal the same values of the radii were obtained; there is practically no distortion due to the weight of the film.

These experiments are well worth repetition; the colors obtained with the rainbow cup are fascinating.

The notion of the atomic constitution of matter, after surviving for more than two thousand years as an element in speculative philosophy, did not attain to the rank of an explanatory hypothesis until roughly a hundred years after the death of Newton. As we shall presently discover, it was the work of Rutherford, early in the twentieth century, which advanced its status still farther, exhibiting it as the very foundation of the whole scheme of interpretation which is the theoretical basis of physical science today.—NORMAN FEATHER, *Lord Rutherford*, 1940.

Binary Numeration before Leibniz

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(Received November 14, 1950)

Though it is frequently stated that binary numeration was first formally proposed by Leibniz as an illustration of his dualistic philosophy, the mathematical papers of Thomas Hariot (1560–1621) show clearly that Hariot not only experimented with number systems, but also understood clearly the theory and practice of binary numeration nearly a century before Leibniz's time.

THE development of the new electronic computing machines has brought to the public eye for the first time the practical application of the binary system of numeration.[1] In this system, every possible number can be written with two arbitrary symbols instead of the conventional ten by expressing all numbers in terms of base two instead of base ten. Usually 1 and 0 are the symbols selected for use, and in this system 2^0 is represented as 1; 2^1 as 10; 2^2 as 100; 2^3 as 1000, etc. For example, the decimal number 1951 represents the symbolic representation of the number in terms of powers of ten: $1\times10^3+9\times10^2+5\times10^1+1\times10^0$; but in the binary system the same number can be expressed in terms of powers of two. In this case, 1951 would be broken down to $1024+512+256+128+16+8+4+2+1$ or in other terms to $1\times2^{10}+1\times2^9+1\times2^8+1\times2^7+0\times2^6+0\times2^5+1\times2^4+1\times2^3+1\times2^2+1\times2^1+1\times2^0$. Using the same indication of "place" in the final representation in the binary system that is used in the decimal system, this number would, thus, be recorded as 11110011111. It is not difficult to see how, under this system, each "place" has but two signifying figures, representing the presence or absence of the power of two in each "place" (the "yes" and "no" of the vacuum tube), instead of the ten required by the decimal system. By the same methods, the same number, decimal 1951, could be represented as 2200021 in the ternary system (requiring three signifying numbers); 132133 in the quaternary system (which requires four); or 30301 in the quinternary system (which requires five). Table I shows how the traditional decimal numbers are translated into binary numbers.

TABLE I. Translation of traditional decimal numbers into binary numbers.

Decimal	Binary	Power
0	0	
1	1	2^0
2	10	2^1
3	11	
4	100	2^2
5	101	
6	110	
7	111	
8	1000	2^3
9	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	
16	10000	2^4
17	10001	
..	.....	
32	100000	2^5
64	1000000	2^6
128	10000000	2^7
256	100000000	2^8
512	1000000000	2^9
1024	10000000000	2^{10}
1025	10000000001	

Mathematical computation, though apparently cumbersome at first, is rendered actually simpler by this method than by the conventional decimal system, since far fewer combinations must be remembered. The fundamental arithmetic operations can be carried out if it is recalled that $1\times0=0$; $1\times1=1$; $1-0=1$; but $1+1=10$. Doubling any number calls for adding a cipher to the number; dividing it by two calls for the removal of a final cipher.

Students of the theory of numbers have generally conceded that though primitive people frequently used binary numeration in their practical dealings, the theoretical consideration of

[1] See for example the popular articles: Harry M. Davis, Sci. American, 28–39 (April, 1949); Popular Sci., 152, (May, 1949).

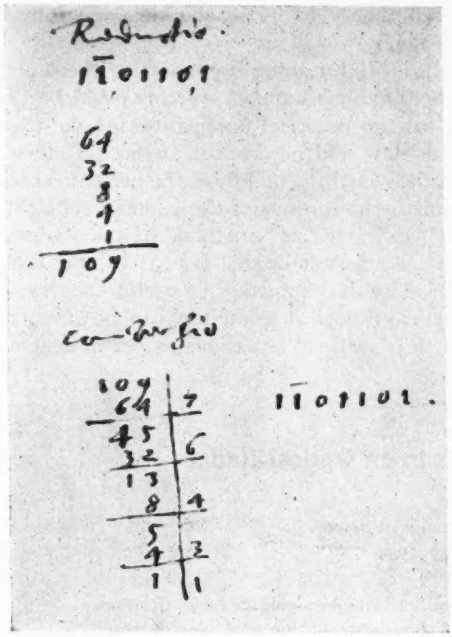

FIG. 1. BM: Additional MSS (Egremont) 6786, f. 346v. Courtesy of British Museum. This shows Hariot's method of conversion from one system of numeration to the other. "Reductio" shows the reduction of a binary number to a decimal number, 1101101 being recorded as $2^6+2^5+2^3+2^2+1=109$; "Conversio" shows the reciprocal process of changing from the decimal number 109 to the binary number 1101101.

such a system came relatively late in the history of mathematics. Almost all are agreed that binary numeration was first formally proposed by Baron Gottfried Wilhelm von Leibniz (1646–1716) as an illustration of his dualistic philosophy, and as mathematical proof that all phenomena of the physical universe could be explained by his assumption of two opposing forces. It is true that Leibniz was tremendously impressed by the neatness of the system which made him exclaim, "*Omnibus ex nihil ducendis sufficit unum*," in direct opposition to the Aristotelian doctrine that nothing could come from nothing. As Laplace wrote,

> Leibnitz saw in his binary arithmetic the image of Creation. . . . He imagined that Unity represented God, and Zero the void; that the Supreme Being drew all beings from the void, just as unity and zero express all numbers in his system of numeration. This conception was so pleasing to Leibniz that he communicated it to the Jesuit, Grimaldi, president of the Chinese tribunal for mathematics, in the hope that this emblem of creation would convert the Emperor of China, who was very fond of the sciences. . . .[2]

Like the majority of the theological-philosophical students of the seventeenth and early eighteenth centuries, Leibniz was willing to find his gods and demons in the formulas of mathematics theory.

The primacy of Leibniz's "discovery" among mathematics theorists is, however, open to question: in fact, it can forthwith be denied. An earlier English scientist, Thomas Hariot (1560–1621), had experimented with number systems, and for his own satisfaction had evolved and considered working with not only binary systems, but ternary, quaternary, quinternary, and higher systems as well. Among Hariot's mathematical papers now in the British Museum, a number of sheets remain to indicate the clarity with which he saw the problems and the facility with which he used the techniques of fundamental calculation in these nondecimal systems. So far

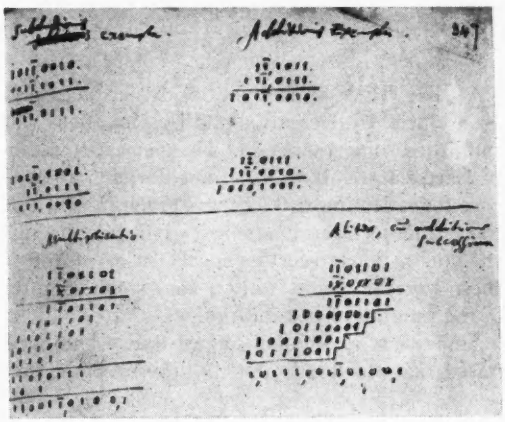

FIG. 2. BM: Additional MSS (Egremont) 6786, f. 347r. Courtesy of British Museum. This figure shows Hariot's manipulation by the binary method. "Subductionis exempla" are two examples of subtraction by binary numbers: the first being $178-59=119$; the second, $169-55=114$. "Additionis Exempla" are two examples in addition: $59+119=178$, and $55+114=169$. "Multiplicatio" shows direct multiplication ($109\times109=11{,}881$) and the same problem is solved "another way, by successive addition" (*Aliter, cum additione successiva*).

[2] Cited by Tobias Dantzig, *Number, the Language of Science* (Macmillan Company, New York, 1939), third edition, p. 15.

as I am aware, no earlier mathematical manuscripts show similar interest in abstract theory of numbers; nor do the extant papers of Hariot's friends indicate that they shared his interest. On the evidence of the manuscript here reproduced (see Figs. 1 and 2), it cannot be asserted that Hariot was the first theoretical mathematician to consider binary numeration for the solution of simple problems. On the other hand, it can definitely be stated that Leibniz was not. Theories of numbers were in the air by the end of the seventeenth century, and it is quite possible that Leibniz merely adapted a currently discussed theory to fortify his own metaphysical philosophy.

Hariot did nothing beyond solve the problem of how various number systems could be used. He saw no practical application for the theory in his day, and he dropped it. So did all other theorists until the middle of the present century, when the limitations of the vacuum tube called for "yes" and "no" methods of calculation. If there is a lesson in this tale, it is that useless knowledge may someday be useful—theory, impractical though it seem, may well be of real and practical value in situations as yet unimagined.

The Forms of Cartesian Ovals in an Optical Range

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(Received January 19, 1951)

This paper supplements an earlier discussion of Cartesian ovals. Assuming for the first medium a refractive index 1.00, and for the second 1.50 (and vice versa), the forms of these ovals for refraction have been studied for numerous cases. Each curve represents the form for a specified value of k, the ratio of image distance to object distance. The values of k run from $+\infty$ to $-\infty$.

A REVIEW of various optical surfaces produced by Descartes and by Huyghens was published some years ago.[1] The general equation of Cartesian ovals was discussed briefly, and the case for reflecting surfaces was given in full. For refracting surfaces it seemed advisable to limit the illustrations to certain special cases, although these were, of course, only a very small fraction of the number of possibilities.

Since then a peculiar sort of query has been raised, one which indicates that the era of molded plastic lenses warrants a set of illustrations of Cartesian surfaces for a much wider range. It is the purpose of this contribution to provide such a review, even though limits must still be imposed.

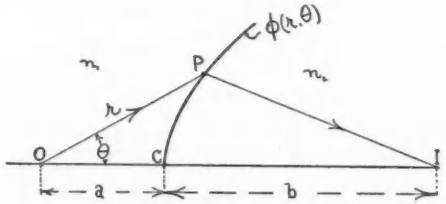

FIG. 1. Refraction of light at a curved surface $\phi(r,\theta)$ separating media of indices of refraction n_1 and n_2. Object and image points are at O and I, respectively. The other symbols are self-explanatory.

A derivation of the general equation was given in the earlier paper; but, to aid in reading the discussion below, the first diagram of that paper is, in effect, repeated here with the equation. Using the symbols shown in Fig. 1, the equation is

$$(n_1^2-n_2^2)r^2-2r[n_1^2a-n_1n_2b-(a+b)n_2^2\cos\theta]+[(n_1^2-n_2^2)a^2+2n_2ab(n_1-n_2)]=0. \quad (1)$$

As shown long ago, this equation reduces to the general equation of a conic when $n_2=-n_1$, that is, for all cases of reflection; otherwise there is still an equation of the fourth degree in Cartesian coordinates. To reduce the problem to one presentable in reasonable space, it may be sufficient to consider only two situations: first, when $n_1=1.00$, and $n_2=1.50$; and second, when $n_1=1.50$, and $n_2=1.00$. These should serve to indicate the type of curves involved for other

[1] Am. J. Phys. 9, 255 (1941).

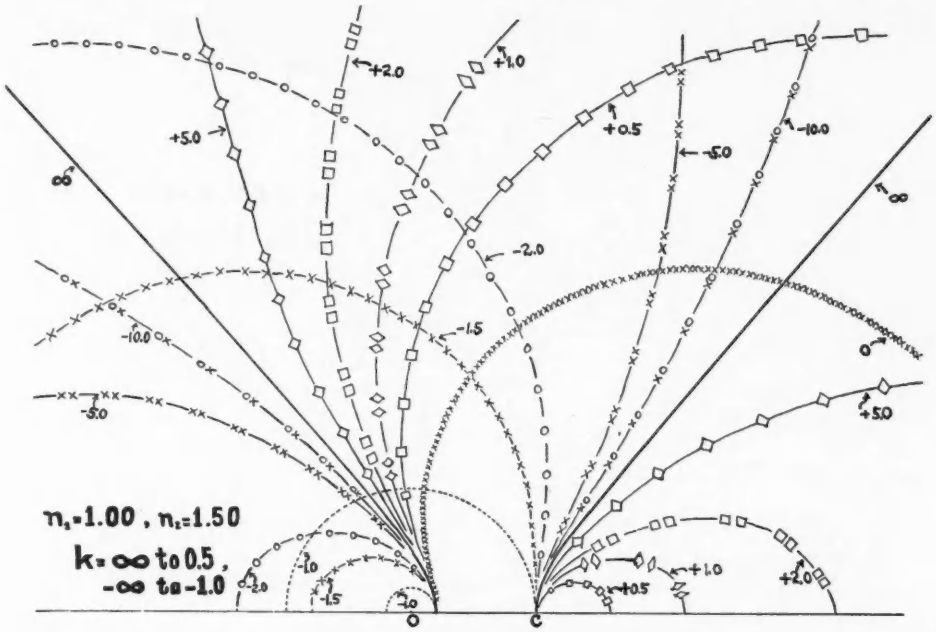

FIG. 2. As in all the figures, the object point is at O, and a is the distance OC. The hyperbola is the curve for $k=\infty$, and the progress of the curve on this diagram is clockwise for positive values of k, and counterclockwise for negative values. The point C is a part of the curve for $k=0$. The two branches become circles at $k=-1$, but this means only that the image is located at the object point.

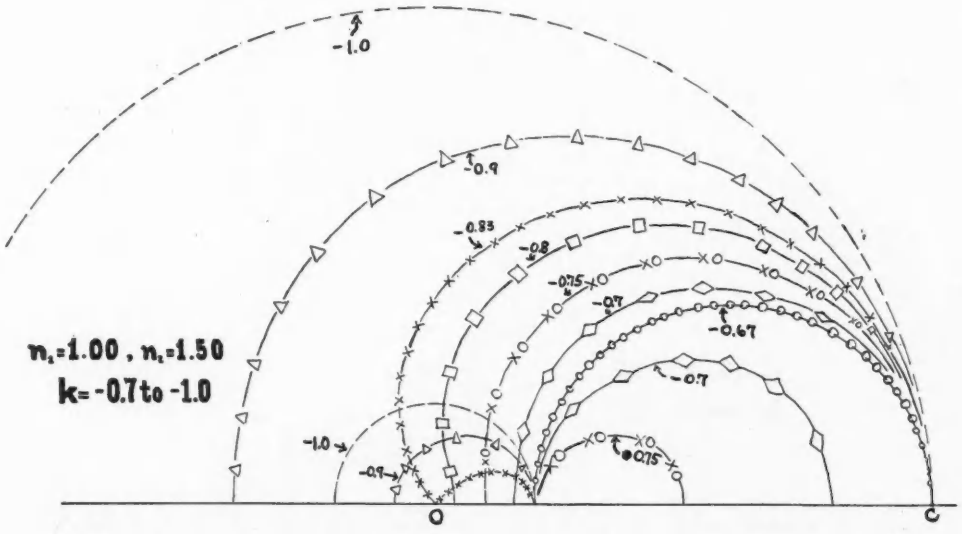

FIG. 3. For this range of values of k, the scale of the drawing is four times that of Fig. 2.

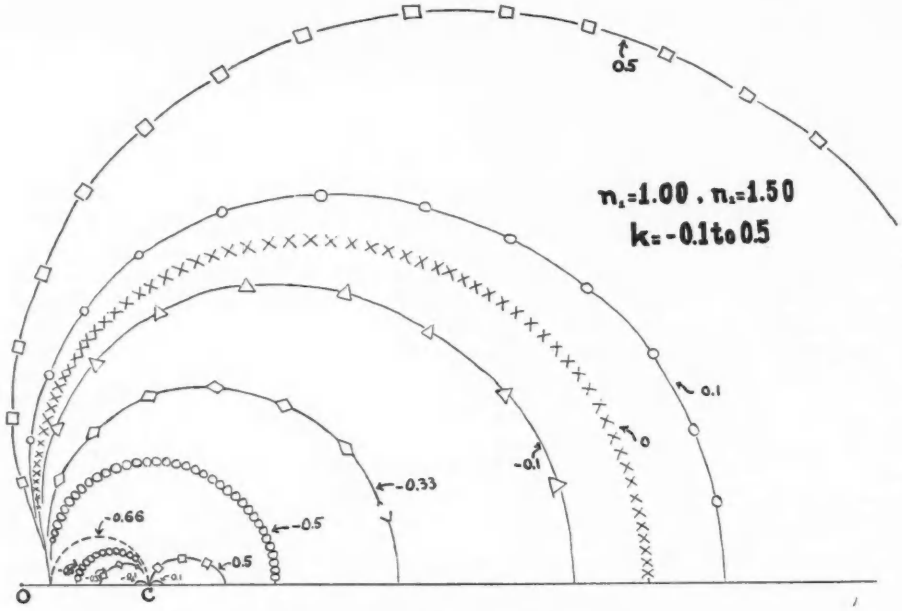

FIG. 4. The scale is the same as for Fig. 2.

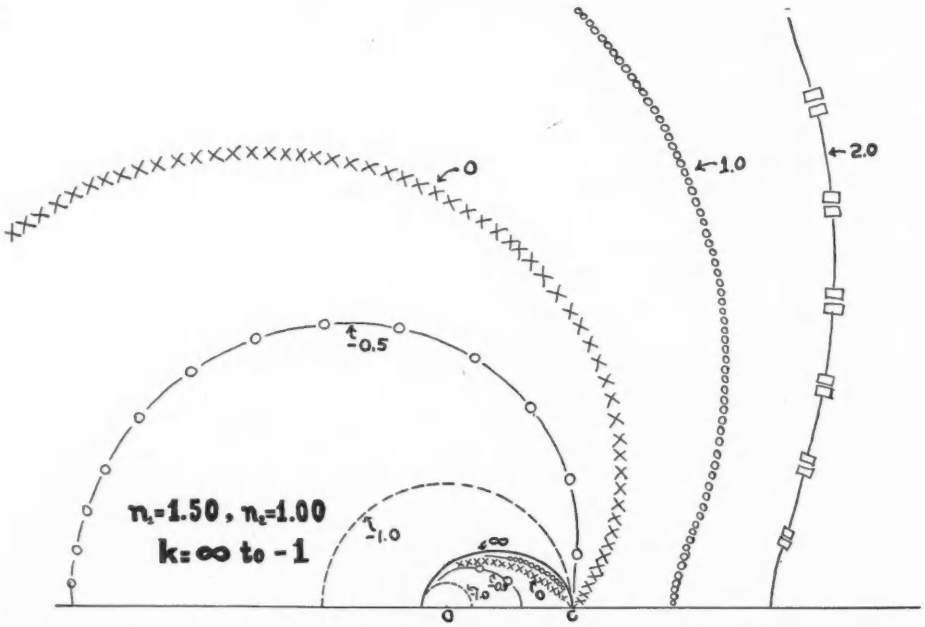

FIG. 5. When the object point is inside the medium of index 1.50, the initial curve for $k = \infty$ is the ellipse shown, with the second branch at infinity. This latter branch moves in rapidly as k takes on smaller values, but the branch inside the ellipse changes so slightly that no attempt has been made to show it before k takes the value 1.0.

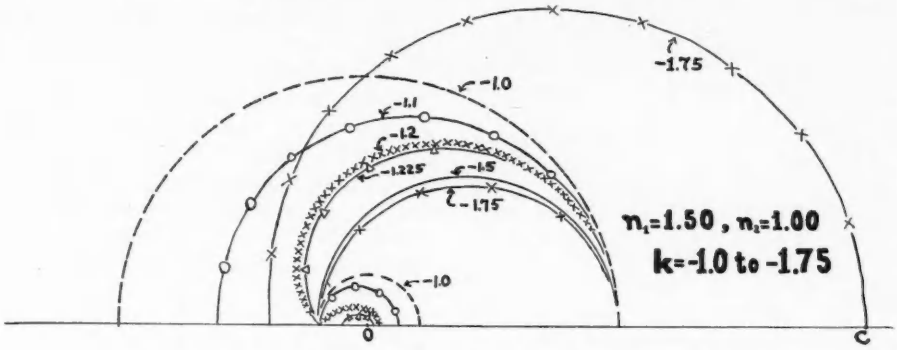

FIG. 6. Like Fig. 3, this is presented on a scale four times that of the others.

indices not too remote from those assumed. For further simplification the image distance will be written as a multiple of the object distance, $b=ka$, where k may have any value from $+\infty$ to $-\infty$.

CASE I. $n_1=1.00$; $n_2=1.50$

The general equation now becomes

$$5r^2+2ar\{(4+6k)-9(1+k)\cos\theta\}+a^2(5+6k)=0. \quad (2)$$

Solving, there results

$$r=-(\tfrac{1}{8}a)[\{(4+6k)-9(1+k)\cos\theta\}\pm\{[(4+6k)-9(1+k)\cos\theta]^2-5(5+6k)\}^{\frac{1}{2}}]. \quad (3)$$

Equation (3) will give the possible forms of the curve when the appropriate values of k are used. For $k=+\infty$, that is, when the image is at infinity, it is simpler to divide Eq. (2) by k before substituting its value. The final result shows the hyperbola $r=a/(3\cos\theta-2)$. As k grows smaller,

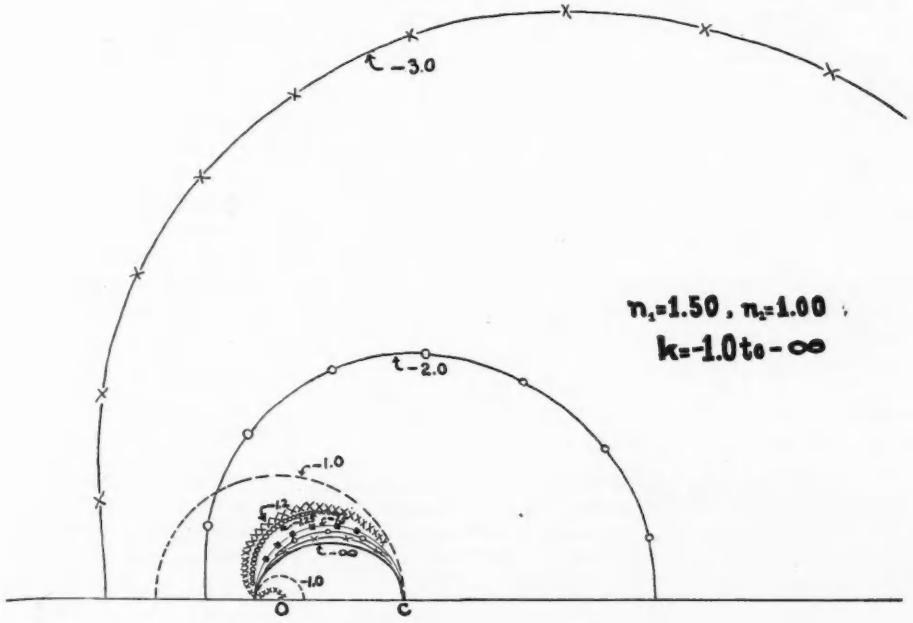

FIG. 7. Here the outer branch goes off the diagram.

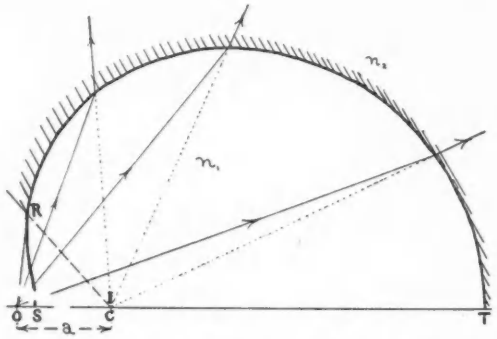

FIG. 8. This is the curve from Fig. 2 when k is zero.

the curve changes in form as shown in Figs. 2, 3, and 4.

Equation (3) indicates that the simpler forms will appear for those values of k which eliminate some complexity therein. For example, if $k=-\frac{5}{6}$, then $r=(\frac{1}{5}a)(1+\frac{3}{2}\cos\theta)$, a limaçon; if $k=-1$, r is independent of θ, and the branches become two circles; and if $k=-\frac{2}{3}$, the two branches coincide in a circle (the case frequently called "Young's construction"). However, for purposes of this paper the diagrams themselves tell the story.

CASE II. $n_1=1.50$; $n_2=1.00$

For these values Eq. (1) becomes

$$5r^2-2ra\{(9+6k)-4(1+k)\cos\theta\}+a^2(5+4k)=0, \quad (4)$$

and the solution is

$$r=(\tfrac{1}{5}a)[\{(9+6k)-4(1+k)\cos\theta\}\pm\{[(9+6k)-4(1+k)\cos\theta]^2-5(5+4k)\}^{\frac{1}{2}}]. \quad (5)$$

Proceeding as before, it is seen that for $k=\pm\infty$, $r=a/(3-2\cos\theta)$, an ellipse. The changes which come with progressive changes in k are shown in Figs. 5, 6, and 7.

While the figures have been planned to conserve space, and yet convey, or indicate, all desired information, it is difficult to show clearly certain interesting features. A single curve will be taken to illustrate the sort of dilemma which may appear when viewing the diagrams. For example, in the diagrams above it is not distinct that in Fig. 2 the point I coincides with the point C when k is zero, yet such is the fact. If $b=ka$, it is obvious that when k is zero, b is also zero, and I must lie on the curve.

Put $k=0$ in Eq. (3) and the result is

$$r=-(\tfrac{1}{5}a)[\{4-9\cos\theta\}\pm\{(4-9\cos\theta)^2-25\}^{\frac{1}{2}}].$$

When $\theta=0$, the values of r are both equal to a, both represented by the point C (Fig. 8). However, as soon as θ increases, r becomes imaginary; hence the point C is the entire inner branch. The outer branch begins to appear when $(4-9\cos\theta)^2=25$ (i.e., when $\cos\theta=-\frac{1}{9}$). Here $r=-a$, and this locates the point R. Successive points follow in both directions from R, until $\cos\theta=-1$, which gives $r=-(\frac{1}{5}a)$, and $r=-5a$, (points S and T).

It should be kept in mind that O, the object point, is in the medium of index $n_1(=1.00)$, and OR is tangent to the curve. Hence the portion of the curve from S to R is meaningless insofar as it suggests that O is in the medium of index n_2. Furthermore, it is easy to show that light in the second medium which has the direction RC meets the surface at R at precisely the critical angle.

There are peculiarities on other curves, but they are similar to the one just described. In any event reference to the equation from which a particular curve is plotted is usually sufficient to clear up a misinterpretation.

There is no such thing as absolute truth and absolute falsehood. The scientific mind should never recognize the perfect truth or the perfect falsehood of any supposed theory or observation. It should carefully weigh the chances of truth and error and grade each in its proper position along the line joining absolute truth and absolute error.—HENRY A. ROWLAND.

On Aerophysics Research*

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(Received December 26, 1950)

This paper deals with the scope of the borderline field between aerodynamics and physics, which has become known as aerophysics. It emphasizes the desirability of utilization of ballistics ranges for aerophysics investigation. In this connection, it reviews in outline form the history of ballistics: the age of speculative ballistics, the age of geometrical ballistics, the age of physical ballistics. Examples of aerophysical research are cited from aeroballistics (high speed gas flows) investigations conducted at the Naval Ordnance Laboratory, White Oak, Maryland.

I. SCOPE OF AEROPHYSICS

THE word thermodynamics is a misleading term, is the paradoxical statement with which E. A. Guggenheim begins his 1949 book on *Thermodynamics*.[1] He cites several illustrations. For example, if a metallic spring has negligible mass, its characteristic frequency depends upon the temperature; hence consideration must be given to the energy flux needed to maintain isothermal conditions during extension and compression of the spring. Strictly speaking, therefore, one may refer to this phenomenon as one of thermodynamics or of thermomechanics. Likewise, the capacitance of parallel plates immersed in a liquid depends upon the temperature. Energy flux, accordingly, is required for the maintenance of isothermal conditions upon charge and discharge. Such a phenomenon more properly belongs to thermostatics. Finally, in the case of a transformer coil immersed in a ferric solution, the mutual inductance depends upon the temperature. Consideration, therefore, must be given to the energy flux necessary for maintaining isothermal conditions upon make and break. This phenomenon is essentially one of thermo-electrodynamics or of thermomagnetism. It is evident that an inclusive term such as *thermophysics* is more descriptive of all these phenomena than thermodynamics.

Analogously, the term *aerophysics* is a better description of high speed gas-flow phenomena than aerodynamics. To be sure, as long as one is interested primarily in drag, lift, and moment of force for low speed bodies in flight, aerodynamics is an appropriate name. The valid approximation of incompressibility, indeed, allows for the equally popular usage of the term hydrodynamics. If, however, one must include the effect of compressibility, which depends upon temperature, one has to consider also energy flux for maintaining isothermal conditions during changes of volume. In this case T. von Kármán's word aerothermodynamics[2] and M. Born's word thermohydrodynamics[3] are more proper descriptions.

It should be borne in mind that thermodynamics *per se* is always associated with physical walls, whether they be adiabatic, diathermanous, or semipermeable. For example, the assumption that acoustic waves are propagated isothermally led early to disagreement between theory and observation. On the other hand, the stipulation of an adiabatic process has yielded satisfactory results for high vibrational frequencies. When the vibrations are slow, however, it is necessary that heat conduction be included in the basic fluid dynamics equations. In general, there are five such simultaneous equations, in five unknowns. It is only in the case of an incompressible fluid that a single thermodynamic equation is adequate, and then only for irrotational flow that a single function suffices. In general, the resulting phenomenological approach to aerothermodynamics is quite complex. What one usually does is to introduce various empirical constants such as specific heats, viscosity,

* Address given at a meeting sponsored jointly by the Midwestern Conference on Fluid Dynamics at the University of Illinois and the Fluid Dynamics Division of the American Physical Society, May, 1950.

[1] E. A. Guggenheim, *Thermodynamics* (Interscience Publishers, Inc., New York, 1949).

[2] T. von Kármán, *Lectures on Aerothermodynamics* (Columbia University Press, New York, 1947).

[3] M. Born, *Natural Philosophy of Cause and Chance* (Oxford University Press, New York, 1949).

thermal conductivity, and diffusion coefficients, which turn out experimentally to be dependent upon the basic variables themselves. It is not surprising, therefore, that M. Born in his 1949 *Natural Philosophy of Cause and Chance* states boldly, "Thermodynamics is misleading." What he emphasizes is that dynamical statements with respect to irreversible transitions are possible only from one equilibrium state to another. In other words, irreversible processes as such are outside the scope of orthodox thermodynamics. Accordingly, there has been an attempt in recent years, initially by L. Onsager,[4] and later by H. B. G. Casimir,[5] by I. Prigogine,[6] and most recently by S. R. de Groot[7] to formulate a new thermodynamics including irreversible processes by introducing an additional assumption, namely, the principle of microscopic reversibility. A clue to a different approach, however, was given by D. Bernoulli in his classical *Hydrodynamica*,[8] where he included the kinetic-theory derivation of Boyle's law together with the now famous Bernoulli theorem of fluid dynamics. Atomism was thus first used as a tool for understanding quantitatively nonequilibrium phenomena. The large number of atoms involved has subsequently made the use of statistical techniques mandatory.

Statistical physics characterizes the state of a dynamical system in terms of its phase, specified by the values of certain dynamical variables. In the usual kinetic theory of gases, for example, there is a density function for each individual dynamical component, whereas in statistical mechanics the density function depends upon all such components. The density function for a single component is determined by solving the Maxwell-Boltzmann integrodifferential equation for an initial value at a particular time. The solution may be obtained in various ways. D. Hilbert[9] first employed an expansion in the neighborhood of equilibrium with infinite molecular density. Other approximate methods have been introduced subsequently by S. Chapman,[10] by D. Enskog,[11] and by D. Burnett.[12] Nevertheless, the mathematical status of the theory is still rudimentary. In recent years analytical work has been generally extended, notably by J. G. Kirkwood[13] in the United States and by M. Born and H. S. Green[14] in Scotland, who use the assumption of molecular chaos. At present, however, it appears that further development along this line must be supplemented by molecular experiments and by experimental mathematics (comparative numerical analysis for various values of parameters).

For complex compressible flow the phenomenological approach, interpreted by statistical physics, may be regarded generally as aerophysics. By this term, then, we understand specifically the study of all phenomena of flowing gases, whether they be in equilibrium or not, whether they be rarefied or condensed, whether they be at high temperature or at low. A typical aerophysical problem is presented by the phenomenon of turbulence. J. Boussinesq (1872) and, more particularly, O. Reynolds (1883) first showed that fluid flow is not usually laminar (as assumed in perfect-fluid theories), but rather an irregular turbulent motion superimposed upon a regular mean motion. Turbulent motion can apparently be described best by statistical methods, suggested initially by O. Reynolds,[15] and later by G. I. Taylor,[16] T. von Kármán,[17] A. N. Kolmogoroff,[18] *et al.* The fundamental understanding of turbulence, however, is still a perennial challenge both to theorists and to experimentalists.

Shock-wave phenomena are equally challenging and illustrative. Let us consider an example of the latter from J. H. Oort's 1946 Darwin lecture[19] "Some phenomena connected with interstellar matter," namely, the head-on collision

[4] L. Onsager, Phys. Rev. **37**, 405 (1931); **38**, 2265 (1931).
[5] H. B. G. Casimir, Revs. Modern Phys. **17**, 343 (1945).
[6] I. Prigogine, *Étude Thermodynamique des Phénomènes Irréversible* (Paris, 1947).
[7] S. R. de Groot, *Thermodynamics of Irreversible Processes* (North Holland, Publishing Company, Amsterdam, 1950).
[8] D. Bernoulli, *Hydrodynamica* (Argentorio, 1938.)
[9] D. Hilbert, Math. Ann. **72**, 562 (1912).
[10] S. Chapman, Trans. Roy. Soc. (London) **A216**, 279 (1916); **A217**, 115 (1917).
[11] D. Enskog, dissertation (Uppsala, 1917).
[12] D. Burnett, Proc. London Math. Soc. **39**, 385 (1935); **40**, 382 (1935).
[13] J. G. Kirkwood, J. Chem. Phys. **14**, 180 (1946).
[14] M. Born and H. S. Green, Nature **159**, 251, 738 (1947); Proc. Roy. Soc. (London) **A188**, 10 (1946).
[15] O. Reynolds, Trans. Roy. Soc. (London) **A136**, 123 (1894).
[16] G. I. Taylor, Proc. Roy. Soc. (London) **A151**, 421 (1935).
[17] T. von Kármán and L. Haworth, Proc. Roy. Soc. (London) **A164**, 192 (1938).
[18] A. N. Kolmogoroff, Compt. rend. acad. sci. U.R.S.S. **30**, 301 (1941).
[19] J. H. Oort, Monthly Notices **106**, 159 (1946).

of two atomic hydrogen interstellar clouds, moving with a relative velocity of about 30 km/sec. If, for simplification, the initial densities and temperatures are the same, the final density of the collision region will be increased threefold and its temperature increased to a point where the hydrogen atoms will be excited and even ionized by collision so that the gas will be luminous. If the initial densities differ, the high temperature region will be confined to a thin luminous cloud, as in the case of Zeta Orionis. Luminosity occurs likewise when a nova like Nova Persei (1901) ejects decelerating shells of different velocities. Similarly, the veil nebula in Cygnus is believed to be the remnant of an expanding supernova.

II. BALLISTICS AS AN AEROPHYSICAL TECHNIQUE

Wind tunnels have been widely used for aerophysical investigations, but ballistics ranges have to a large degree been neglected for this purpose. In view of the fact that ballistics has become somewhat of a lost academic art, it may not be inappropriate to review the milestones of its historical development.

The art of throwing stones is probably the oldest of the military sciences; for example, one finds a Biblical reference to it in the reign of Uzziah (*ca* 800 B.C.): "And he made in Jerusalem engines, invented by cunning men, to be on towers and upon the bulwarks, to shoot arrows and great stones withal."[20] Undoubtedly, such engines were primitive ballista (βαλλειν, to throw), which were used later to hurl primarily darts and arrows. The Roman catapult (κατα, down +παλλειν, to hurl) was another such device. (In this connection, it will be recalled that Archimedes, the great mathematical physicist, was engaged in such wartime scientific investigations about the third century B.C.) The French trébuchet, a medieval modified machine, was based upon the principle of the lever. Out of these military operations there developed considerable philosophical interest in the underlying mechanisms. The first historical period may be called the age of speculative ballistics.

Speculation was particularly rife with respect to the propulsion of missiles. For example, Aristotle in the fourth century B.C. believed that a stone or arrow in flight is propelled because of the air pressure behind it—because of the air in front of the body displaced to its rear. It is noteworthy, however, that the Alexandrian, Joannes Philoponos,[21] in his commentary on Aristotle's *Physica* stated that flight will take place equally well in a vacuum. Nevertheless, the Aristotelian concept prevailed until modern times. The only other outstanding insight is ascribed to the Franciscan Friar William of Ockham,[22] who in his fourteenth-century commentary on Aquinas' *Five Ways* suggested that propulsion should be characterized by a nonmaterial cargo (perhaps akin to our modern concept of momentum). It is for this reason, I believe, that E. T. Whittaker in his 1949 *From Euclid to Eddington*[22] regards "Ockham as the chief founder of modern science." Regardless of the merits of this particular observation, it is certainly true that his times mark the beginning of a quest for dynamical principles.

There has always been considerable speculation also as to the actual path of a missile. It is quite surprising to find in the sixteenth century a practical gunner like Santback believing that a missile travels in a straight line and then immediately drops vertically. Other more ingenious, philosophically minded individuals regarded the path as a combination of the elements of a straight line and of a circular arc (for the terminus). It was only as occasional individual like Nicolo of Brescia (nicknamed Tartaglia—the stammerer) who drew an actual sketch of "the way of a pellet" that was not a regular geometric curve such as a parabola. It is not surprising that in the matter of propulsion there was confusion, but it is truly amazing that so much misunderstanding could exist as to the form of the path, inasmuch as the latter is more susceptible to ordinary observations.

The age of speculative ballistics was followed by the age of geometrical ballistics. In Galileo's 1638 *Dialogues Concerning Two New Sciences*[23]

[20] II Chronicles 26:15.

[21] M. R. Cohen and E. Drabkin, *A Source Book in Greek Science* (McGraw-Hill Book Company, Inc., New York, 1948), p. 217.

[22] E. T. Whittaker, *Space and Spirit* (Regnery, Hinsdale, 1948); *From Euclid to Eddington* (Cambridge University Press, London, 1943).

[23] G. Galileo, *Dialogues Concerning Two New Sciences* (translation-Northwestern University, Evanston, Illinois, 1939).

there is a discussion of the sciences of motion, including so-called violent motions such as those of a projectile. Galileo had perceived the roles of inertia and of the acceleration due to gravity. Accordingly, by combining the horizontal effect of inertia with the vertical one of gravitational acceleration, he deduced that the path of a horizontally projected object in a vacuum is a parabola. What he failed to appreciate, however, was the disturbing influence of the medium itself. Thus in the fourth book[23] he says, "Among these projectiles which we make use of, if they are of a heavy matter and a round form; nay if they are of a lighter matter and have a cylindrical form, such as arrows shot from bows, their track or path will not sensibly decline from the curve of a parabola." His successor, E. Torricelli, merely extended Galileo's proof to bodies projected at any angles.

It is historically significant that the very enthusiasm for the theoretical mechanism that was thus initiated as a new science by Galileo evoked the same irrational loyalty as had the previous philosophical speculations. Thus the mechanistic deduction of a parabolic trajectory became venerated in the same fashion as Euclid's axiomatic proofs about earth measurements—regardless of actual experience. For example, in 1683 Blondel wrote a book,[24] *L'Art de jetter les Bombes*, which was awarded the 1667 French Academy prize for a practical scheme "to hit a mark," without mentioning the retarding effect of the medium. As a matter of fact, geometrical, mechanistic reasoning was so intrenched in the minds of intellectuals that, in 1707, the Secretary of the French Academy remarked[25] that nothing was then wanting for gunnery, except possibly instrumentation. Even as late as 1717 practical artillerists were censured for not using the geometrical principles, despite the fact that de Resons[26] stressed the inadequacy of such principles for correcting the failures to hit the mark. All this popular misunderstanding, however, was abetted by leading scientists of the time. For example, Anderson in his 1674 *Genuine Use and Effects of the Gun* had already pointed out that a parabolic path is not in agreement with observations. Unfortunately, he ascribed the discrepancy to a hypothesis that was itself contrary to the laws of motion. Even E. Halley[27] in his 1686 *A Discourse Concerning Gravity* said, "The principles (except for small and light shot) may be put into practice to all intents and purposes as if this impediment (air resistance) were absolutely removed." On the other hand, hand, in his 1687 *Principia* Sir Isaac Newton[28] showed that the path of a body in a resisting medium can theoretically be considerably different from a parabola. What is more, his experiments on the resistance of air in St. Paul's Cathedral, London, verified that the effect of air resistance is not negligible in practice. C. Huygens[29] in his 1690 *Discourse of the Cause of Gravity* mentioned his own Paris experiments as a confirmation of the Newtonian idea that actual trajectories are not parabolic. Nevertheless, the dogmatism of geometrical mechanics prejudiced practical men about ballistics until the beginning of the eighteenth century.

The modern period, which may be called the age of physical ballistics, begins with the classical, experimental work of B. Robins.[30] In his 1742 *New Principles of Gunnery* he gives the results of the first actual determination of the velocity (about 1700 ft/sec) of a projectile by means of his ballistics pendulum. Moreover, by using different distances between the points of projection and of observation he was able to determine approximately the drag function (the increase over the velocity-square law was found to begin at about 400 ft/sec). These experiments were continued by C. Hutton with larger weights. Sir John Pringle[31] in his 1778 *Discourse on the Theory of Gunnery* said: "He [Robins] justly concluded that the foundation here was at least as much an affair of physics as of geometry."

The next significant contribution to physical ballistics did not occur until toward the close of

[24] Blondel, *L'Art de jetter les Bombes* (Paris, 1683).
[25] Hist. acad. roy. sci. A(1707).
[26] Mém acad. roy. sci. A(1716).
[27] E. Halley, Phil. Trans. No. 179 (1686).
[28] I. Newton, *Mathematical Principles* (translation, University of California Press, Berkeley, California, 1946), Book II, Section 7.
[29] C. Huygens, *Discours de la Cause de la Pesanteur* (Leyden, 1690).
[30] B. Robins, *New Principles of Gunnery* (Hutton edition, London, 1850).
[31] J. Pringle, *A Discourse on the Theory of Gunnery* (London, 1778).

the nineteenth century. E. Mach[32] in his 1897 popular lecture, "On some phenomena attending the flight of projectiles," tells how his interest in the physics of ballistics was aroused during a Paris lecture given by a Belgian artillerist Melsens. The latter had claimed that a projectile carries before it compressed air, which is responsible for the great damage done upon impact. Mach resolved to search for experimental evidence of this more dense air by use of the Töpler scheme (Foucault knife-edge test for optical striae, plus Huygens' addition of a telescope and a spark) for detecting acoustic waves. He was surprised to find no evidence at all of any compression, until he realized that the particular projectile was traveling subsonically (at about 250 m/sec). Accordingly, he repeated the experiments, under the sponsorship of the Austrian Naval Department, at a higher speed and then detected a head wave of compressed air. He and his son, L. Mach, later conducted similar experiments on a large scale at the Krupp works and on a small scale in the Prague University Laboratory. (It is interesting that these so-called schlieren observations were made prior to the development of the shadowgraph technique by C. V. Boys in 1895.) The primary contribution of Mach, however, was the discovery that the compression of the air at the front of a projectile is due not to mass motion, but rather to wave motion. In this connection, he utilized an interferometer to measure quantitatively the density in front of the head wave and behind it, and thus found that the change of density is insufficient to support the damage attributed by Melsens. The following remark of Mach in his lecture is not without significance today: "Research of the kind that we are considering can certainly not be detrimental to practical artillery, and it is no less certain that experiments by artillerists will undoubtedly be of benefit to physics."

Mach's insight as to the usefulness of optical techniques for investigating aerophysical phenomena has been restricted so far largely to practical wind-tunnel tests. It was not until World War II that the interferometer became even recognized, both here and abroad, as a potential instrument for aerophysical research. In this case, too, attention has been focused upon its use in wind tunnels rather than in ballistics ranges. Nevertheless, it is important to realize that firing ranges are necessarily complementary facilities for such investigations. As a matter of fact, the divergence between aeronautics and ballistics has become less as the speeds both of aircraft and of projectiles have increased. Figure 1(a) illustrates qualitatively the historical development of missile speeds. In Fig. 1(b) there is a comparison of the evolution of the configurations of missiles and of aircraft. One sees that modern supersonic projectiles have essentially taken on wings and that modern supersonic aircraft have become more pointed, like projectiles. Accordingly, the trend to a common form suggests a common region of aeronautics and ballistics interests. It has become customary to speak of this field of research and development as *aeroballistics*. Whether from the standpoint of supersonic and hypersonic wind tunnels or from that of ballistics ranges, aeroballistics affords techniques for the study of aerophysical phenomena—as Mach predicted.

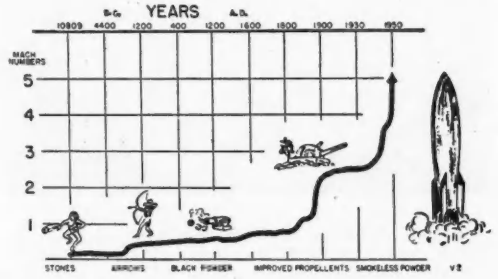

FIG. 1(a). The increasing Mach number of missiles.

[32] E. Mach, *Popular Scientific Lectures*, translated by T. J. McCormack (Open Court Publishing Company, LaSalle, Illinois, 1943), fifth edition.

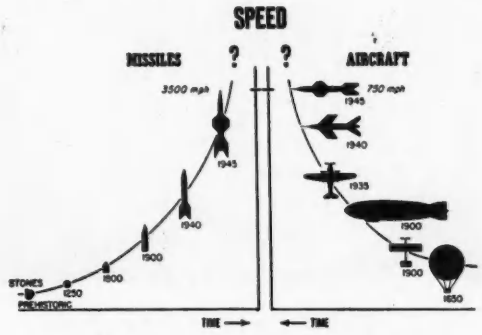

FIG. 1(b). Projectile and aircraft configurations.

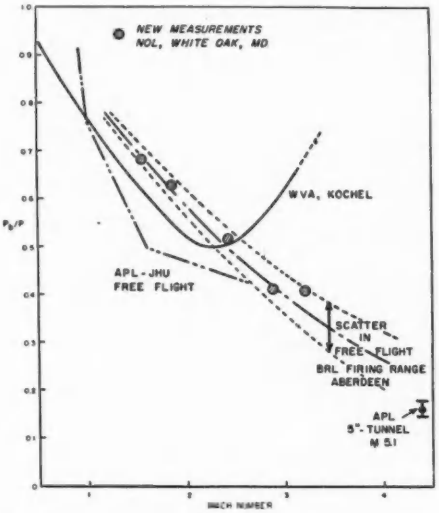

FIG. 2. Comparison of wind tunnel. Free flight results. Base pressure *versus* Mach number.

III. AEROBALLISTIC RESEARCH AT THE NAVAL ORDNANCE LABORATORY

Aeroballistic research can be carried out both experimentally and theoretically. Theoretical investigations are being greatly facilitated by the development of high speed calculating machines, which make possible modern experimental mathematics. Aeroballistics problems can be investigated experimentally, in wind tunnels, in which a model is stationary, and in ranges, in which the medium is stationary. Of course, the size of the models used will determine the scaling information that is finally required for practical adaptation. The Naval Ordnance Laboratory (NOL) has underway such a balanced, comprehensive program of aeroballistic research, with which I am personally acquainted. Attention is directed here, however, to certain fundamental problems and techniques that involve the use primarily of the NOL ballistics ranges. In each instance preliminary results are cited for illustrative purposes and not as complete analyses.

Whereas supersonic flight has been available to ballisticians for a long time, supersonic wind tunnels are of comparatively recent origin. In general, wind-tunnel and range results on the drag of slender bodies agree well up to about a Mach number of 2.5. H. H. Kurzweg (NOL),[33] however, has noted that not only is there a discrepancy at higher Mach numbers (see Fig. 2),

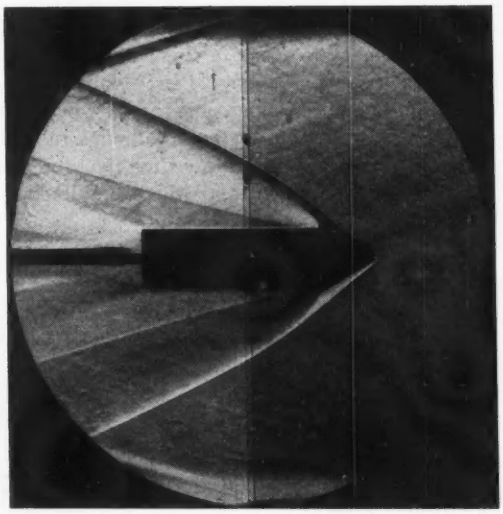

FIG. 3(a). Wind-tunnel schlieren photograph of a cone-cylinder with a laminar boundary layer—Mach number 3.24; Reynolds number 1.3×10^6.

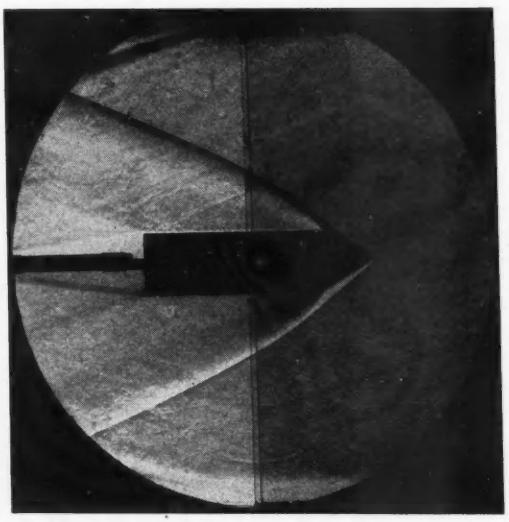

FIG. 3(b). Wind-tunnel schlieren photograph of a cone-cylinder with a turbulent boundary layer (same as in Fig. 3(a) except for slightly modified cone)—Mach number 3.24; Reynolds Number 1.3×10^6.

[33] H. H. Kurzweg, "Interrelationship between boundary layer and base pressure," to be published in J. Aeronaut. Sci. (1951).

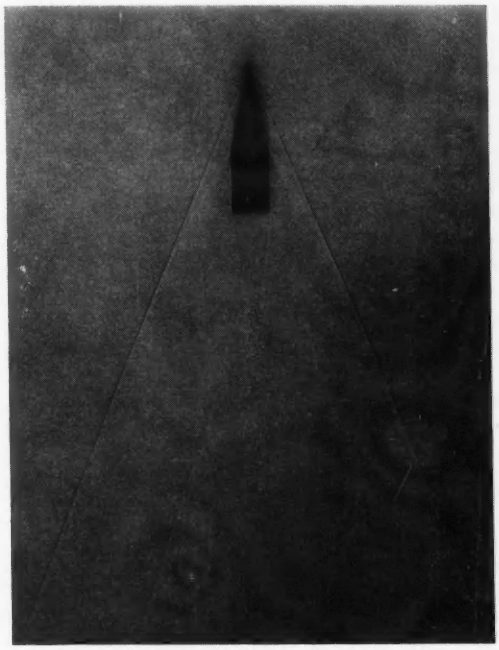

FIG. 4(a). Shadowgram of a 0.30-caliber cone cylinder fired into the NOL ballistics range (air pressure 5.41 inches of mercury)—laminar boundary layer—drag coefficient 0.218—Mach number 3.067; Reynolds number 0.387×10^6.

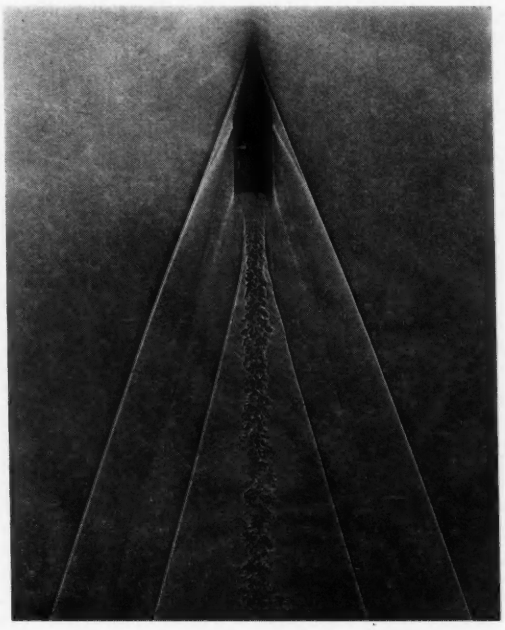

FIG. 4(b). Shadowgram of a 0.30-caliber cone cylinder fired into the NOL ballistics range (air pressure 30 inches of mercury)—turbulent boundary layer—drag coefficient 0.246—Mach number 3.37; Reynolds number 2.36×10^6.

but that wind-tunnel results actually give a lower drag because of base pressure, and that range results show a higher drag. Later investigations at NOL indicate that the source of the discrepancy can be traced primarily to the nature of the boundary layer. It appears the boundary layer of the models used in previous wind-tunnel investigations was laminar, whereas that in free flight was turbulent. Both types of boundary layers were reproduced in an NOL aeroballistics wind tunnel by making small modifications in the model. The qualitative differences, seen in Figs. 3(a) and 3(b), account quantitatively for the discrepancy. Accordingly, it is evident that wind-tunnel and range results must be used in a complementary fashion.

The Reynolds number, which measures essentially the inertial effect relative to that of the viscous force, plays an important role in subsonic scaling. A question arises as to whether or not the same usefulness exists at supersonic speeds. Unfortunately, the answer is complicated by the fact that the highest Reynolds number which can be obtained at present in wind tunnels is approximately 20 million, whereas large guided missiles can have Reynolds numbers of 150 to 200 million in free flight. The NOL Pressurized Ballistics Range is a facility that goes a long way toward bridging this gap. It consists essentially of a tube 319 ft long and 3 ft in diameter, with 25 spark stations and with 7 timing channels; space can be determined to approximately 0.5 mm, and times measured to less than 0.2 microsecond. The temperature is constant, the humidity is controllable, and the pressure is variable from above 6 atmos to below 0.01 atmos; Reynolds numbers are possible up to 50 million. Preliminary investigations by D. Shanks (NOL) show that at a Mach number of 1.7 the transition from laminar to turbulent boundary layer on a specific cone-cylinder missile takes place at a pressure of between $\frac{1}{6}$ atmos and $\frac{1}{3}$ atmos, whereas at a Mach number of 2.5 the transition occurs between pressures of $\frac{1}{3}$ atmos and 1 atmos. Thus, the flow pattern for a given

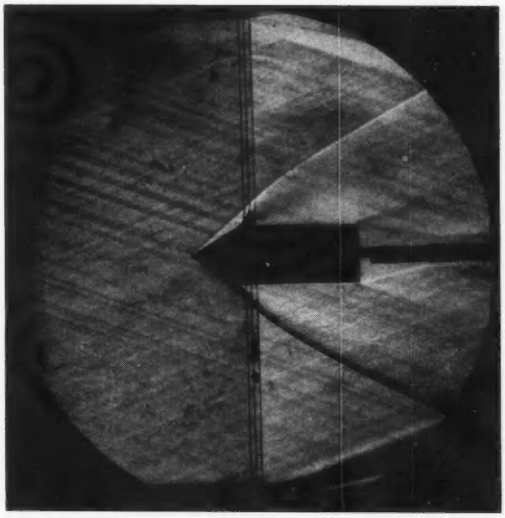

FIG. 5(a). Wind-tunnel schlieren photograph of a cone-cylinder at 40°F—base pressure 40 percent of free-stream pressure—Mach number 2.86.

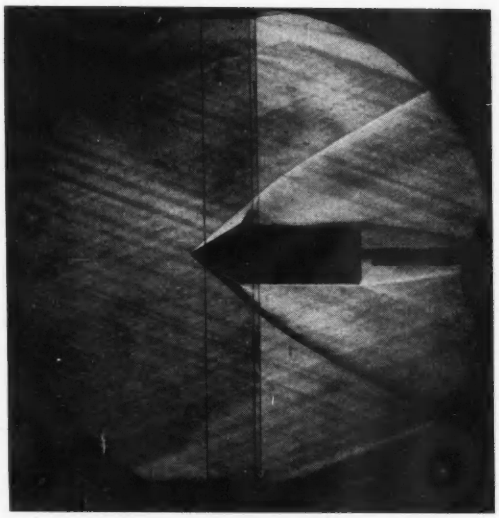

FIG. 5(b). Wind-tunnel schlieren photograph of a cone-cylinder at 400°F—base pressure 62 percent of free-stream pressure—Mach number 2.86.

Mach number depends critically upon the Reynolds number (see Figs. 4(a) and 4(b). This problem is being pursued further.

T. von Kármán[34] first pointed out that the Reynolds number may be resolved into two factors, namely, the Mach number, which measures effectively the compressibility, and the Knudsen number, which measures effectively the degree of breakdown of the physical continuum. Accordingly, if either factor becomes meaningless (i.e., excessively high or low density), the Reynolds number concept itself may become inadequate for describing scaling phenomena. Thus, the relationship between compressibility and viscosity may be expected to become more complicated as speeds increase. A typical phenomenon is the interaction of a shock wave and a boundary layer in supersonic flow. As still higher speeds are attained, however, another phenomenon becomes equally important, namely, thermal effects. The Peclet number, which is the similarity parameter of a flow field with superimposed heat and which measures essentially the convective effect relative to that of conduction, can be resolved into two factors, namely, the Reynolds number and the Prandtl number, which measures the relative importance of viscosity and thermal conductivity. The Peclet number, too, may lose its value as a scaling parameter. As the Reynolds number and Mach number become undefined, therefore, it is to be expected that at a high Mach number the whole matter of flow will become more complicated. Preliminary investigations along this line have been carried out by H. H. Kurzweg.[33] Naval Ordnance Laboratory wind-tunnel experiments show that for the same Mach number and Reynolds number the flow pattern and resulting drag depend essentially upon the temperature profile in the boundary layer (see Figs. 5(a) and 5(b)).

The nature of the flow in a wind tunnel or in a ballistics range differs, moreover, from the flow relative to a full-scale missile in flight. One of the significant parameters is turbulence, which must be investigated in free streams and in boundary layers, including heat transfer and mixing processes. Only fragmentary results in this research program have been achieved to date. For example, F. N. Frenkiel (NOL) has shown analytically that the statistical description of time fluctuations and of space fluctuations of isotropic turbulence in a wind tunnel are not *a priori* the same. Using high speed calculating

[34] T. von Kármán, Z. angew. Math. Mech. **3**, 395 (1923).

machines, he has computed the time-correlation curve and verified this conclusion numerically.

The introduction of heat at high speeds of flow introduces additional problems. At present, comparatively few observational results are available, owing to the experimental difficulty of achieving sufficiently high speeds. In this connection, J. H. McMillen (NOL)[35] has produced speeds up to 7150 ft/sec for a ⅛-in. sphere in air (see Fig. 6(a)). Figure 6(b) shows such a sphere entering into water. In the latter case, the optical phenomena are particularly complex owing to the pressure fields created (on the basis of similar investigations H. Schardin has suggested that a change of state occurs). By firing a 0.22-caliber projectile at a speed of 7000 ft/sec in cold air, P. A. Thurston (NOL) has attained a Mach number of 9.6 (see Fig. 6(c)). Still higher speeds have been observed by J. F. Stanton of the Naval Ordnance Test Station, who observed the particles in the jet of a shaped-charge liner at a speed of 20,000 ft/sec. With these higher speeds one is rapidly drawing near to the region of meteors, which travel at a Mach number of about 30.

The phenomena which occur at high speeds in free flight represent complex physical processes from the standpoints both of thermodynamics and of kinetic theory. For example, in aerodynamics one usually assumes the equation of state and the internal-energy function of a so-

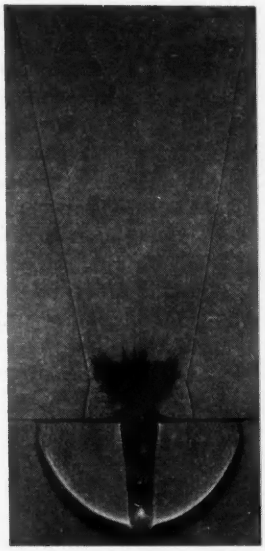

FIG. 6(b). Shadowgram of a ⅛-in sphere fired into water at a speed of 6870 ft/sec.

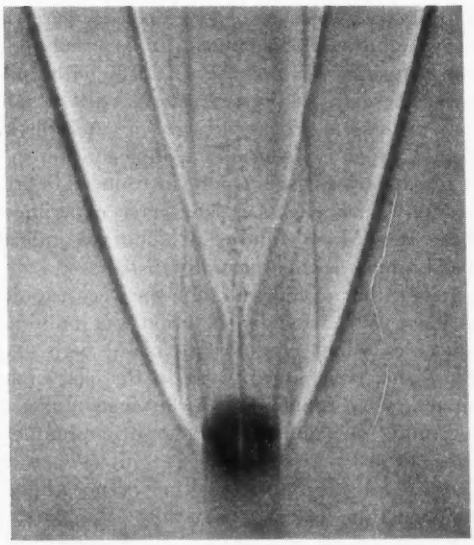

FIG. 6(a). Shadowgram of a ⅛-in. sphere fired in air —Mach number 6.3.

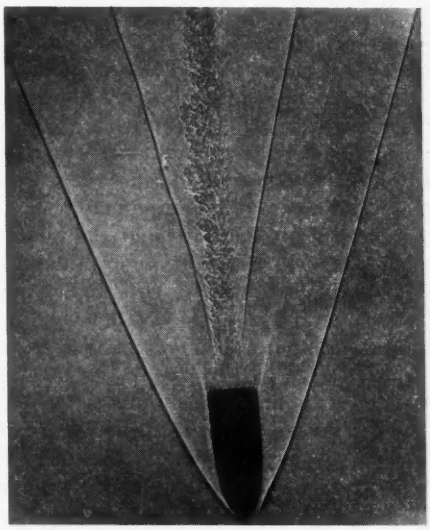

FIG. 6(c). Shadowgram of a 0.22-caliber projectile fired through air at nearly liquid-air temperature—Mach number 9.6.

[35] McMillen, Kramer, and Allmand, J. Appl. Phys. **21**, 1341 (1950); R. L. Kramer and J. H. McMillen, Phys. Rev. **73**, 1255 (1948).

called ideal gas. Calculations by J. C. Crown (NOL) show that for a shock wave, traveling at a Mach number of 7 in air at room temperature, changes in the equation of state are relatively unimportant, whereas utilization of a different energy function (involving temperature-dependent specific heats) will produce considerable differences in the final temperature. In the case of an isentropic expansion starting from different initial temperatures but from a constant initial pressure (50 atmos), and expanding to a Mach number of 5, he found that the final pressure differs little from that obtained for the ideal-gas approximation. A further study of the combined effect of such isentropic flow through a normal shock wave shows there is a considerable change in temperature, but relatively little change in pressure, in comparison with ideal-gas results. At high Mach numbers, therefore, measurements of pressure and of force would apparently not be particularly sensitive to variations in the thermodynamic factors.

For a gas rapidly expanding from high temperatures to low ones, there is always a possibility that time will not permit the establishment of kinetic equilibrium. In this case, the instantaneous values of the specific heats may correspond to fewer degrees of freedom than those of the final state. The resulting difference in the ratio of specific heats for a single vibrational state has been shown by K. F. Herzfeld and F. O. Rice[36] to account for the observed dispersion of ultrasonic waves. R. N. Schwartz[37] (NOL) has further extended the calculation of L. Landau and E. Teller[38] to include the effect of relaxation when several modes of vibration may be excited. Preliminary results indicate that the relaxation times may be of considerable importance in high speed flow phenomena. At still higher speeds there may occur also dissociation and even ionization (compare detection of meteor trails by radar).

With growing theoretical interest in higher and higher speeds and with developing experimental techniques, aerodynamics itself has become more and more a matter of aerophysics. Whereas not so long ago quantum mechanics was considered to be a mysterious domain, sacred to the high priests of theoretical physics, nowadays aeronautical engineers daily have business to transact there. Indeed, whenever the energy involved in the collision between a missile and an atmospheric molecule becomes sufficiently high, the probability of excitation and even of dissociation of nitrogen, oxygen, and other constituents of air becomes appreciable. From the aerodynamicist's point of view, the fluid apparently assumes new properties. The behavior of the gas is described no longer by classical, but by quantum laws. For example, the rate at which energy is transferred from translational energy of the missile into internal energy of the gas is a problem in the quantum mechanics of collisions. In aerodynamics this rate of transfer will appear as a shock thickness. (It is conceivable that under certain circumstances the entire missile will be imbedded in such a transition region.) Accordingly, aerodynamic coefficients such as drag, stability, and heat transfer will be quite different; they will now be functions of a new parameter, namely, the Debye temperature. Physical understanding becomes of paramount importance. Moreover, it is then evident that these high speed phenomena can be simulated at low velocities by the use of gases with much lower energies. At present, the Naval Ordnance Laboratory is constructing an Aerophysics Range, which is essentially a single-station tube, in which various gases can be introduced in order to study such effects. Preliminary experiments there at a Mach number of about 13 indicate that bromine is then partially dissociated.

It is rather significant that the very problem of energy transfer by collision of individual molecules, which is important in the high speed free flight, is significant also in the case of hypersonic wind tunnels. In the latter instance, the flow may be below the condensation temperatures of the constituents (oxygen, nitrogen, *et al.*) The critical question is whether or not there is sufficient time for condensation to take place. In either case, the nature of the fluid changes so that the problem of analyzing data becomes still further complicated. P.P. Wegener[39] (NOL) has

[36] K. F. Herzfeld and F. O. Rice, Phys. Rev. **31**, 691 (1928).
[37] R. N. Schwartz, Phys. Rev. **82**, 298(A) (1951).
[38] L. Landau and E. Teller, Physik. Z. Sowjetunion **10**, 34 (1936).

[39] P. P. Wegener, "Summary of recent experimental investigations in the NOL hyperballistics wind tunnel", to

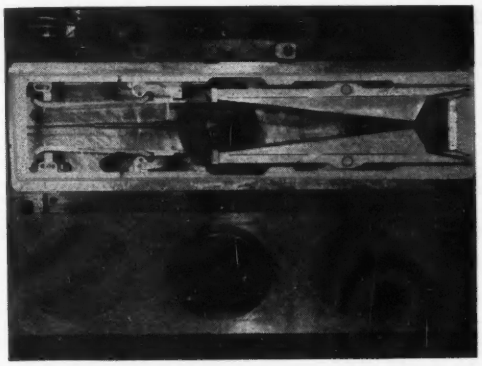

FIG. 7(a). Working section of NOL 12×12 cm hyperballistics wind tunnel No. 4.

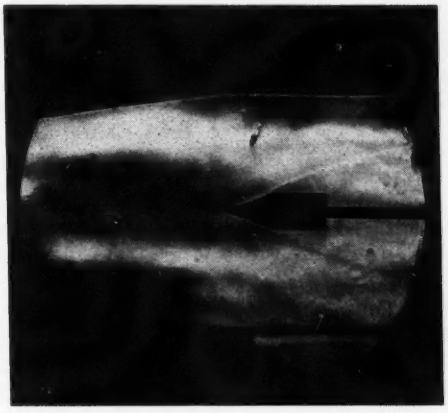

FIG. 7(b). Wind-tunnel schlieren photograph of a 40° cone cylinder—Mach number 8.25 (no condensation).

been conducting a number of investigations both in the NOL hyperballistics wind tunnel (see Figs. 7(a) and 7(b)) and in a shock tube to ascertain the conditions under which such condensation takes place. To date, the highest Mach number attained by him in a flow without condensation (preheated) is 8.25.

Thus, the high speed flow of a gas runs a gamut of problems from high temperatures and pressures to low temperatures and pressures. All these problems are essentially physical, they involve not only thermodynamics, but also kinetic theory including quantum-mechanical considerations. It seems that modern aeroballistic research, both in wind tunnels and ranges, offers a real means of investigating aerophysical phenomena. The excellent physical investigations of Mach having lain dormant for many years, it would be well for us all, particularly teachers of physics, to wonder more at phenomena that have been ignored as commonplace. In this respect, M. von Laue's statement[40] in his 1950 *History of Physics* is well worth pondering: "Philipp von Jolly told the inquiring young Planck that physics was essentially worked out and that pursuit of this science could hardly be profitable."

be published in J. Aeronaut. Sci. (1951); P. P. Wegener and G. Lundquist, J. Appl. Phys. **22**, 233 (1951).

[40] M. von Laue, *History of Physics* (New York, 1950).

Science professes to eliminate "here" and "now." When some event occurs on the earth's surface, we give its position in the space-time manifold by assigning latitude, longitude, and date. We have developed a technique which insures that all accurate observers with accurate instruments will arrive at the same estimate of latitude, longitude, and date. Consequently there is no longer anything personal in these estimates, in so far as we are content with numerical statements of which the meaning is not too closely investigated. Having arbitrarily decided that the longitude of Greenwich and the latitude of the equator are to be zero, other latitudes and longitudes follow. But what is "Greenwich"? This is hardly the sort of term that ought to occur in an impartial survey of the universe, and its definition is not mathematical. The best way to define "Greenwich" is to take a man to it and say: "Here is Greenwich." If some one else has already determined the latitude and longitude of the place where you are, "Greenwich" can be defined by its latitude and longitude relative to that place; it is, for example, so many degrees east and so many degrees north of New York. But this does not get rid of "here," which is now New York instead of Greenwich.—BERTRAND RUSSELL, *Human Knowledge*, 1948.

Are X-Ray Tube Demonstrations Safe?

R. SCHLEGEL AND J. C. LEE
Michigan State College, East Lansing, Michigan
(Received February 5, 1951)

Measurements of the radiation in the neighborhood of an unshielded demonstration x-ray tube give intensities ranging from 6 roentgens per minute within a few cm of the tube to no appreciable radiation per minute a few meters from the tube. The genetic and body effects of radiation are discussed. These effects are such that x-ray demonstrations should be given with greater care than is perhaps now generally exercised.

THE demonstration of an operating x-ray tube is customarily included among the lecture demonstrations of a general physics course. Besides showing how the radiation makes a mineral-coated screen fluoresce and demonstrating the absorbing characteristics of different inanimate materials, the instructor often has several members of the class place their hands between the x-ray tube and the fluorescent screen to show the relative x-ray shadowing powers of bone and other tissue. Demonstrations such as these are undoubtedly a valuable part of the student's introduction to the subject of x-rays. We have wondered, however, whether there is a possibility of students or instructor receiving what may be considered to be excessive amounts of radiation in the course of such demonstrations. In this paper we first report on measurements we have made on the radiation intensity at various points in the neighborhood of an unshielded x-ray tube. The results obtained will then be discussed in the light of some recent statements about the biological effects of ionizing radiation.

APPARATUS AND PROCEDURE

A standard cold-cathode demonstration tube was used as the source of x-rays. The electrical potential difference applied across the tube was supplied by an ordinary induction spark coil activated by a six-volt Edison storage battery. The potential difference applied to the tube from the secondary of the coil was measured and found to be 70 kilovolts peak. The tube current was about 2×10^{-4} ampere.

The radiation intensities were measured in roentgens, using a standard Victoreen r-meter. The reader will recall that a roentgen (abbreviation: r) is that amount of ionizing radiation which will produce one electrostatic unit of ions (2.08×10^9 ion pairs) in one cubic centimeter of dry air at 760 mm of mercury and 0°C. We would claim no more than order-of-magnitude accuracy for our measurements, and our results have therefore been rounded off to a single significant figure for the times of exposure indicated.

No shielding of any sort was used. The central glass sphere of the tube (Fig. 1) is approximately 9 cm in radius, and the target is at the center of the sphere. Positions at which intensities were measured are indicated as numbers of centimeters from the tube, such measurements being made from the glass wall.

RESULTS

Our results from measurements in the neighborhood of the tube are (see Fig. 1):

I. Along horizontal line A, which is perpendicular to the electron beam and which is at 45° with the face of the target, radiation intensities of about 6 r per min in a range of distances from 2 cm to 20 cm were measured. At 30 cm from the tube, a decrease to 4 r per min was noted.

II. At 20 cm from the tube along the horizontal lines B and C, which are at angles of 0° and 90°, respectively, to the target face, we found a somewhat smaller intensity on B than on C; but in both cases no striking decrease from the intensity of 6 r per min noted in item I.

III. Along A', which is line A extended back through the target and tube, we found at 2 cm from the tube wall an intensity of 0.4 r per min. This radiation is undoubtedly scattered radiation from the tube wall.

IV. Along D, a horizontal line extending

through the target from the target center, one r per min was measured at 2 cm. This intensity, too, must be ascribed chiefly to scattered radiation.

V. A measurement was made at a point 23 cm from the tube along *A*, but 28 cm vertically below the line *A*. This point in our arrangement was at the edge of the lecture demonstration table, and might well be a point at the gonad or lower visceral region of a person working near the table. We found an intensity of 0.5 r per min at this point. Measurements at points 25 cm to the right and left of this point along the table edge and parallel to the x-ray tube length gave about the same intensity, though with some decrease at the point to the left (toward the projection on the table top of the line *B* in Fig. 1).

VI. A measurement was made at the writing tablet level on a student's front seat in our lecture room. This point was 230 cm from the tube along *C*, but 20 cm below *C*. We found that 5 minutes operation of the tube gave no detectable reading on our meter at that point.

VII. We also measured the intensity at a point 230 cm from the tube, along the horizontal line *C*, but directly in front of a room wall (painted cinder block). At this point the intensity was found to be 0.04 r per min. The difference from the reading in item VI is to be ascribed, of course, to secondary radiation scattered from the wall.

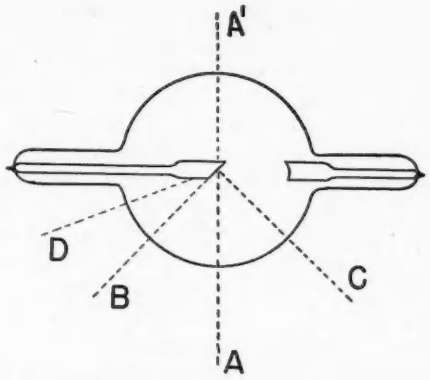

FIG. 1. Sketch of the x-ray tube, with reference lines for indicating the positions at which intensity measurements were made. The lines *A*, *B*, *C*, *D* converge on the target where the x-rays originate.

DISCUSSION

The known effects of radiation on living organisms have been recently discussed in two papers by H. J. Muller, winner of the Nobel prize for his work in genetics.[1] We shall rely on Muller's papers for biological information that will enable us to make some judgment on the hazards of the radiation intensities which we have found to accompany lecture room x-ray demonstrations. We recommend the papers by Muller to the interested reader for a detailed account of the evidence supporting various conclusions of Muller which we shall use.

The exposure of a student or instructor to a few r of radiation during an occasional demonstration would not be considered a harmful dose to the body by ordinary medical standards. Such exposure might, however, have undesirable genetic consequences, and, if received fairly regularly by an instructor, could apparently have untoward effects on his or her own life.

Muller points out that relatively small radiation doses, persistently applied to an organism, will apparently result in a reduction of life-span.[2] The evidence, gained from tests on mammals of widely differing natural life spans, indicates that for human beings each ten r of radiation per year, *applied every year*, will cause a shortening of the life span of the individual by one year. The radiation intensities which we found in the neighborhood of a demonstration x-ray tube are such that a physics instructor who gave such demonstrations several times a year and who worked rather close to the tube for a few minutes during each demonstration would very likely receive 10 r or more during each teaching year.

Muller points out, incidentally, that the accepted tolerance dose of 0.1 r per day for research and industrial workers who are exposed to radiation of any kind involves a maximum exposure of about 30 r per year. This exposure, received year after year, "will probably shorten their lives by three years."[3] A somewhat less positive statement is given by Shamos and Roth,

[1] H. J. Muller, Am. Scientist **38**, 33 (1950); **38**, 399 (1950). These papers will be referred to as I and II, respectively.

[2] Reference 1, II, pp. 416–417.

[3] Reference 1, II, p. 417.

but none the less with the recommendation that the tolerance dose, in accordance with current tendency, be reduced from 0.1 to 0.05 r per day.[4] It is interesting to note that cosmic rays give at sea level a radiation dose of about 0.03 r per year. At an elevation of 16,000 ft the yearly exposure would amount to about 0.3 r. The biological effects of an r of high energy cosmic radiation might be considerably different, however, from the effects of an r of x-ray radiation.[5]

The genetic effects of radiation—by which we mean the effects on the descendants of the exposed individual—are discussed by Muller chiefly on the basis of experiments which have been performed on fruit flies. The validity of his conclusions rests on an extrapolation of his results to higher forms of life, including man. Experiments with mice justify such an extrapolation to mammals. In the absence of exact data on human beings it seems reasonable, in agreement with Muller, that "the only defensible policy is to frame our practices on the basis of the already existing data from lower forms."[6]

The genetic effects of radiation occur through the mechanism of radiation-induced mutations in the gene structure of germ cells. Experiments indicate that the number of such mutations at any one time is directly proportional to the total past radiation which an organism has received in the gonad region. The number of mutations is proportional to the total roentgen dose, without regard for the intensity at which that dose was received. The radiation-induced mutations represent an increase over the naturally occurring spontaneous mutations. Specifically, the number of radiation-induced mutations in 1000 germ cells is approximately equal to the radiation, in roentgens, to which the cells have been exposed.[7]

Among 1000 first-generation offsprings of organisms which have been exposed to 150 r of radiation there will be found about 150 individuals that carry radiation-induced mutations. Muller estimates that in a given generation of offspring, much fewer than 25 of the 150 mutations will be dominant lethal mutations. About 25 of the mutations will be recessive lethal, which means that they will be carried in the line of descendants until, on combination with lethal recessives from another parent, they result in the death of an offspring. The remaining of the 150 mutations are in general "recessive detrimentals," which will be carried by successive descendants until matched with like detrimentals from another parent, whence they will then give rise to some kind of manifest biological deficiency—detrimental but not lethal—in an offspring.[8]

The striking aspect of the effects of radiation, as presented by Muller, is the tragic inevitability of those effects. Individuals whose germ cells have received one r of radiation must expect that among each 1000 descendants there will be, on a statistical average, one individual with a resulting genetic deficiency which will appear from time to time as a biological handicap in various descendants and will eventually be one-half the cause of a "genetic death."

The radiation-induced mutations are, as stated, in addition to naturally occurring mutations which are with some degree of persistence being carried from generation to generation. We cannot present here Muller's detailed arguments about the relative effects of radiation-induced and spontaneous mutations, but instead we quote his conclusions about the effects to be observed among the first generation descendants of individuals whose germ cells have been exposed to 150 r of radiation before reproduction. In this first generation an increase of only about 10 percent in hereditary abnormalities would be observed. "This slight increase would, however, be repeated in the second, third, and subsequent generations, and would only gradually fade away in the course of very many generations as the induced mutant genes were eliminated extremely slowly by reason of the somewhat higher than normal death rate, or the somewhat lower than normal fertility, of the individuals carrying them."[9]

[4] See M. H. Shamos and S. G. Roth, *Industrial and Safety Problems of Nuclear Technology* (Harper and Brothers, New York, 1950), pp. 251–253. The older tolerance standard of 0.1 r per day, and the recent revision to half that amount, are reported in the *Eighth Semiannual Report of the Atomic Energy Commission* (United States Government Printing Office, Washington, D. C., July, 1950), pp. 11–12.

[5] See V. F. Hess and J. Eugster, *Cosmic Radiation and Its Biological Effects* (Fordham University Press, New York, 1949), from which the values given for cosmic radiation have been adapted.

[6] Reference 1, I, p. 56.

[7] Reference 1, I, p. 40.

[8] Reference 1, I, p. 42.

[9] Reference 1, I, p. 48.

Muller further estimates the effect of each individual of our society receiving an average radiation dose of 100 r during his lifetime, with this dose rate continuing generation after generation. This is not an exaggeration, Muller opines, aside from possible warfare exposures, if present protection standards and present trends in usage of radiation sources continue.[10] The "steady state" in gene mutations, in a population receiving the 100-r lifetime dose per individual, would entail, according to Muller's estimates, a doubling in the number of deaths resulting from gene mutations, and also, a "doubling of the genetic or constitutional causes of ailments and weaknesses of all sorts."[6] This multiplication of expressions of detrimental mutations "would be a very serious situation indeed, and life might hardly seem worth living for many persons."

CONCLUSION

In view of Muller's expert biological opinion—with which biologists seem to have no quarrel—it would appear that we physics teachers should indeed exercise care in our use of radiation sources. Inasmuch as our measurements indicated that no considerable amount of radiation reaches the students' desks in an ordinary lecture room arrangement, it is perhaps within the bounds of prudence to give a brief demonstration of an x-ray tube to each class. The bringing of a student close to the tube, however, so that he may see his hand bones outlined on a screen, hardly seems to be permissible, particularly in view of the fact that our students usually have their reproductive periods before them. Instructors, too, who may give hundreds of demonstrations in a teaching career, would seem to be well-advised to stay well away from an operating tube. In our own lecture demonstrations, since becoming aware of the radiation intensities near a tube we have set up remote controls several yards removed from the tube. Discussion of radiation hazards and precautions at this point adds a good bit, we find, to the interest and liveliness of the topic of x-rays.

[10] Reference 1, I, p. 55.

Scientists and Mobilization

(Abstracted from the report of the Science Advisory Committee of the Office of Defense Mobilization, Oliver E. Buckley, Chairman.)

Questions . . . arise as to what are the particular steps that may be taken by universities or individuals seeking more direct participation in defense research. Each situation must, of course, be judged on its own merits, and institutions and individual scientists must arrive at their own decisions, but as a general guide the Science Advisory Committee has the following suggestions:

1. Outstanding men should be made available to systems projects of the type referred to above and to other defense activities where, in performing their function as part of a group, they can learn the research needs by participating in determining them.

2. Specially qualified leaders should be given leaves to take positions in the Department of Defense and other government agencies which will familiarize them with the problems most needing attention.

3. Universities and colleges should welcome opportunities for joint projects of special importance where the scientists who participate can give full-time attention to the problems they undertake.

4. Individuals should themselves seek opportunities where they can serve best. To a considerable extent the best place for an individual scientist may depend on whether his own interest and aptitude are for fundamental research and teaching or for applied research or administrative work in defense projects.

5. Periods of leave for such defense work should be of specified duration so that individuals will not be too long separated from their normal activities. This will result in participation by a greater number of individuals and more widespread benefits.

6. In determining actions in this connection, the importance to the nation of research and teaching is a major consideration. In this we concur with the view already expressed by the Chairman of the Atomic Energy Commission and the Secretary of the Navy that "The scientist in his laboratory and the research professor with his graduate students are performing a service which may make a critical difference to our country in the difficult years ahead."

NOTES AND DISCUSSION

A Simple Method for Locating Principal Points

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THE focal length and the location of the principal points of a thick lens or lens system can be determined with moderate precision and a minimum of equipment and calculation by a method shown in Fig. 1. It is particularly applicable to convergent systems; a somewhat similar but less elegant scheme can be adapted to divergent systems. A point source L' is placed on the principal axis and another such source L is placed y cm off axis; as an alternative one can shine a light through two small holes at L and L'. A diaphragm D or a hole of small size allows two narrow pencils to fall on the lens. By adjusting the position of D one can make the emergent pencils parallel; their parallelism is shown by a fixed separation h as the screen S is moved to and fro. A piece of graph paper or a rule affixed to the screen facilitates detection of convergence or divergence and also allows h to be measured. Then the distance x, from L' to the focal plane D, is all that is needed in order to find the focal length from the relation $f=hx/y$, and thence the location of the principal plane P. Determination of the correct position for D can also be made by focusing the image of a distant object to the right of the system.

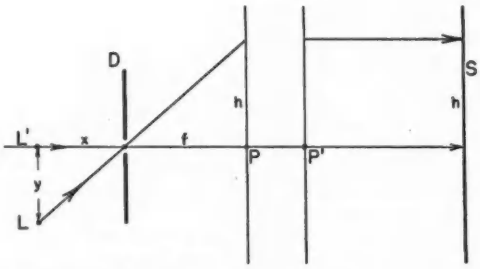

FIG. 1. Sketch of method of finding principal points of an optical system.

Inspection of the geometry will show that the above equation still holds even if L' and/or the center of diaphragm D are somewhat off the principal axis. Thus precise setting on the axis is not necessary but it does facilitate observation of the parallelism of the emergent rays.

This arrangement is usable in an intermediate optics experiment on thick lenses. Thick lenses can be made up by cementing plano-concave or plano-convex thin lenses to opposite sides of a rectangular glass block, or even to the ends of Lucite cylinders of various lengths. Some demonstrations performed with such lenses are shown and described in Grimsehl's *A Textbook of Physics*, Vol. 4, *Optics* (Blackie and Son, Ltd., London, 1933), p. 84 ff.

An Experiment for the Direct Measurement of Magnetostatic Fields

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MANY textbooks define the magnetic induction B in terms of the force on a current-bearing conductor. A simple experiment in which this force is actually used to make a direct measurement of the field is therefore desirable.[1] The experiment suggested here has proven successful in an electricity and magnetism laboratory course and has also been useful as a lecture demonstration.

A wire stirrup or loop is mounted on a horizontal axis between the poles of a permanent magnet whose field is directed vertically downward. When a current i is passed through the stirrup it will rotate through an angle θ to a new position as shown in Fig. 1. On the element of length s the sideways force $F=isB$ deflects the wire stirrup from its normal vertical position, whereas the forces on the wire elements of length b and a give rise to no torque about the axis of rotation. The torque due to the magnetic force is equated to the torque due to gravitational forces in Eq. (1).

$$isBb\cos\theta=\frac{mgb(b+s)\sin\theta}{(2a+2b+s)}. \tag{1}$$

Here m is the mass of the stirrup and g is the acceleration due to gravity. Solving for B one obtains

$$B=\frac{mg(b+s)\tan\theta}{s(2a+2b+s)i}. \tag{2}$$

The quantities on the right are easily measured and thus B is determined.

The details of the actual apparatus and circuit are shown in the figure. The magnet, with pole pieces of approximately

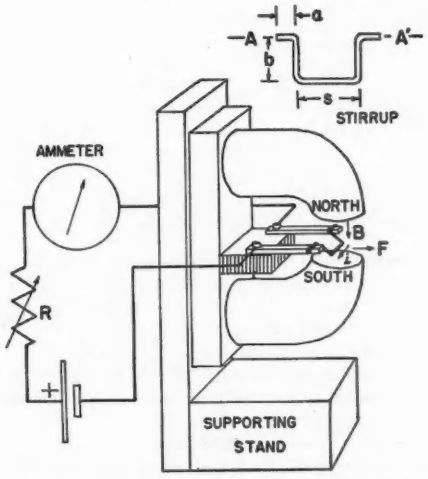

FIG. 1. The wire stirrup is supported between the poles of the magnet. The magnetic force is determined from the deflection of the stirrup.

1½ inches diameter spaced about 1½ inches apart, is an ordinary magnetron magnet with a field of the order of 2500 gauss. The wire stirrup is supported on two metal bearings insulated from each other to which the external circuit is connected. The currents required depend on the diameter and length of the copper wire used as the stirrup, but are of the order of one or two amperes for a deflection of 45° when the wire is 0.040 in. in diameter. The angle θ is easily measured by lining up the two sides of the stirrup with a transparent protractor. In practice there may be some difficulty because of erratic contact at the bearing. This can be overcome by making the contact very smooth or by providing a *very* light spring pressure holding the wire against the bearing.

In the experiment the students first weigh and measure the stirrup and then record the angle θ for several different currents. In this way data are obtained for computing the magnetic field using Eq. (2). In a second part of the experiment the magnetic field obtained in this way is compared with that obtained in the well-known experiment with a flip coil and a ballistic galvanometer.

The experiment described here is not highly accurate, but it does give the student a clear understanding of the force on a current-bearing conductor in a magnetic field. The force is strong enough so that he can push on the wire with his fingers and actually feel it.

[1] See also, Harvalik, Am. J. Phys. **19**, 128 (1951), and the experiments of Ampère described in W. F. Magie's *Source Book of Physics* (McGraw-Hill Book Company, Inc., New York, 1935), p. 446.

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LETTERS TO THE EDITOR

Bonus Questions and Recommended Reading

RECENTLY, much work has been done in the development of courses which emphasize the methodology and philosophy of science, mostly by using historical and "block-and-gap" patterns of organization. However, these courses are mainly for nonscience majors and although one would like to provide the same opportunities for students in the conventional premedical and engineering courses, this is not always possible for administrative or other reasons.

The following scheme was tried this last year in a standard premedical course and found to be quite successful, although it is admittedly only a partial solution to the problem. At the beginning of each semester, the students were told that a certain book was "recommended" outside reading for the course and that each examination would include "bonus questions" based on this book so that those who had read it should be able to improve their over-all grades. A perfect score on each examination was nominally 100, but enough bonus questions were included so that the maximum possible score was increased to 115. It is not very difficult to devise questions which can be answered fairly easily only by those who have done the reading thoughtfully. The books recommended were *Language in Action* by Hayakawa and *The Evolution of Physics* by Einstein and Infeld; other choices are certainly possible depending upon the tastes and objectives of the instructor.

The question "Why were you asked to read this book in a physics course?" was given as the bonus question in the final examination of the first semester in order to test the results of the method. The answers to this question and comments made by students during conversations showed that the objectives of the instructor—that they learn the importance of clear and careful definitions, the methods of abstraction, and the construction and validation of concepts in science—had been achieved to a surprisingly high degree. In addition, students commented that they liked Hayakawa's book because of the new ideas it introduced them to, and that they "should have been required to read it while freshmen."

ROALD K. WANGSNESS
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Fitch's Apparatus for the Measurement of The Thermal Conductivity*

THE Cenco-Fitch Heat Conductivity Apparatus[1] has been designed for student use for quick measurements of the thermal conductivity of poorly conducting materials with reasonable and reproducible accuracy at low cost. The quick determination allows an experiment on the thermal conductivity of a poor conductor to be performed in a normal laboratory period, which the constant-flow type of experiment usually does not.

If greater accuracy is desired than is provided by the apparatus as it is, correction must be made for the thin air films on both sides of the sample and the loss of heat from the copper receiving block. The heat loss may be corrected for by Newton's law of cooling.[2] The former correction may be made by the use of the following relation, easily proven.

$$l/K = l_1/K_1 + l_2/K_2 + l_3/K_3 + \cdots,$$

where K is the experimentally obtained value of the thermal conductivity; l_1, the thickness of the air gap (determined by using small shims to space the source and receiving blocks); K_1, the known thermal conductivity of air; l_2 and K_2, the length and true thermal conductivity of the sample; l_3 and K_3, the length and thermal conductivity of the second air gap, etc.; and $l = l_1 + l_2 + l_3 + \cdots$.

FRANK P. FRITCHLE
Central Scientific Company, Chicago, Illinois

* J. Satterly, Am. J. Phys. **19**, 132 (1951).

[1] A. L. Fitch, Am. Phys. Teacher **3**, 135 (1935). Worthing and Halliday, *Heat* (John Wiley and Sons, Inc., New York, 1948), p. 182. Central Scientific Company, Chicago, Cat. J-150, p. 1167.

[2] A similar correction is described on p. 132, Edwin Edser, *Heat for Advanced Students* (Macmillan and Company, Ltd., London, 1944).

Concerning Classroom Recitation

PROFESSOR Dodd's elegant exposition on "The Old-Time Classroom Recitation: Can It Be Restored?"[1] emboldens me to report a practice which I have pursued in recent years and which meets the problem in a partial way at least.

We are well agreed that in our modern instructional scheme there exists hardly a single device in which the student gets training in self-expression. Recitation in class, that is, answering questions put by the teacher, does not provide it. This degenerates invariably to verbatim repetition of the textbook. Quizzes do not provide it, even though we may now and again make them expositional. Multiple-response or problem quizzes make no contribution whatsoever to literary expression. The laboratory provides extremely limited occasion for a continued and connected oral recitation. As Dodd puts it, "the recitation should be a lecture in miniature." This is precisely what my scheme is designed to accomplish.

After some 10 or 12 weeks of the first semester, when the student has acquired a little backlog in physics, I announce that we will have "brief lectures" by members of the class on topics of interest to them. These need not be textbook subject-matter, but they can be. If the topic is taken from literature elsewhere it is restricted to the physics already covered in the course. The subject is cleared with me and the student indicates roughly what he plans to embrace. Late in the second semester the choice of material is obviously richer. The student now possesses greater physics competence and the "new physics," we find, offers an attraction. Sometimes a biographical sketch is given with emphasis on the physics, and it is invariably well done. The student stands at the blackboard, chalk in hand, and lectures. The presentations run 10 to 15 minutes. Questions may be put from the floor. In short, *the student gives a lecture.*

Since my classes are invariably small we can get around to a student at least twice a semester. The first time up is a matter of great misgiving for some, but we are all aware of the very substantial benefits accruing. Students have reported that this aspect of the physics course appears to them the most valuable one.

JULIUS SUMNER MILLER

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New Orleans, Louisiana

[1] L. E. Dodd, Am. J. Phys. **19**, 14 (1951)

The Mathematics of Elementary Thermodynamics

I HAVE recently noted what appears to be an error in the article "The Mathematics of Elementary Thermodynamics" by Professor Menger.[1] In the example of a mole of ideal gas in Sec. 2(*b*), it is stated that

$$u-u_0=c_v(\theta-\theta_0),$$

if the volume is constant, and that

$$u-u_0=c_p(\theta-\theta_0),$$

if the pressure is constant. Since the internal energy of an ideal gas is customarily assumed to be a function of temperature only, the first equation should apply equally well for a constant pressure process (or for any other process whatever which takes the gas from an initial temperature θ_0 to a final temperature θ).

With this correction, we have

$$\Delta q=\int_{v_0}^{v_1}\frac{R\theta_0}{v}dv+\int_{\theta_0}^{\theta_2}c_v d\theta+\int_{v_1}^{v_2}p_0 dv+\int_{\theta_2}^{\theta_3}c_v d\theta.$$

The result is

$$\Delta q-p_0\Delta v-c_v\Delta\theta=R\theta_0\ \ln(v_1/v_0)-p_0(v_1-v_0),$$

which is the same as given by Menger except that the factor two does not appear in the last term. Using a power series expansion for the logarithm, we may obtain

$$\Delta q-p_0\Delta v-c_v\Delta\theta=p_0v_0\ln\left[1+\frac{v_1-v_0}{v_0}\right]$$
$$-p_0(v_1-v_0)=-\frac{p_0v_0}{2}\left[\frac{v_1-v_0}{v_0}\right]^2+\cdots.$$

Thus we see that, while the "correction term" does not approach zero with Δv and $\Delta\theta$, it does become a second-order term in (v_1-v_0).

This may be seen qualitatively as follows: The so-called "correction term" is simply the heat added during a cycle when v_3 is put equal to v_0. This must equal the work done during the cycle. The latter is, of course, equal to the area of the loop on the $p-v$ diagram. We might therefore expect this to be a second-order term in (v_1-v_0) as the cycle becomes very small.

This error does not invalidate the conclusion which the author draws from this example, nor detract in any way from the principal theme of the article.

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[1] Karl Menger, Am. J. Phys. **18**, 91 (1950).

The Mathematics of Elementary Thermodynamics

MR. Thomsen[1] is right in pointing out the error in my paper[2] and also in saying that the "correction term" is of second order in v_1-v_0. With regard to the latter remark, however, I wish to emphasize that v_1 is merely an auxiliary quantity, and that, in my opinion, the real question is how my corrective term behaves in relation to v_3-v_0 and $\theta_3-\theta_0$, (the difference between initial and end parameters) rather than in relation to v_1-v_0. Now in relation to the difference between initial and end parameters, the corrective term (even after it has been corrected according to Mr. Thomsen) is *not* of second order nor even of *first* order. It is of *zero* order; that is to say, it need not be smaller than the difference in question. In fact, it may even be very large compared with it. This might be mentioned in order to forestall mistaken inferences from Professor Thomsen's remark.

Moreover I wish to mention that in a letter[3] to me of June 7, 1950, Professor M. W. Zemansky pointed out the same error that Professor Thomsen has found.

Finally, in this connection, I wish to say that, always due to the same error, also in the second column of p. 92 and on p. 93 of my original paper, the c_p ought to be replaced by c_v with the effect that the factor $-2\rho_0$ is to be replaced by the factor $-\rho_0$.

KARL MENGER

Illinois Institute of Technology, Chicago, Illinois

[1] J. S. Thomsen, Am. J. Phys. **19**, 476 (1951).
[2] K. Menger, Am. J. Phys. **18**, 91 (1950).
[3] Private communication.

On Stirring a Cup of Tea

WITH respect to the inquiry of Professor Miller[1] about the motion of sugar crystals towards the center of the bottom of a cup of stirred tea, it may be noted that the accepted explanation was first given nearly a century ago and appropriately enough, by the Briton, James Thomson.[2] His explanation is recounted and accepted by Sydney Goldstein[3] in a more general discussion of fluid flow in curved channels, wherein the meandering of streams is accounted for by the same mechanism. The phenomenon depends upon the retardation of the lower layers of fluid by viscous drag against the bottom of the cup. In the upper layers of fluid, the pressure distribution is such as to preserve a pressure gradient varying linearly outward from the center in order to give the accelerating centripetal force; this pressure gradient is preserved through the lower layers of fluid, but the centripetal force resulting therefrom is greater than that needed to provide the smaller acceleration in the more slowly moving lower layers of fluid. Hence, there is a motion of the fluid from the outside of the bottom regions towards the center, with entrainment of the contained solids; the fluid rises at the center and is carried outward and downward along the walls of the cup.

D. J. MONTGOMERY

Textile Research Institute, Princeton, New Jersey

[1] J. S. Miller, Am. J. Phys. **18**, 534 (1950).
[2] James Thomson, British Association Reports, Dublin, 1857, Trans. of Sections, p. 39; Papers in Physics and Engineering. p. 147.
[3] S. Goldstein, *Modern Developments in Fluid Dynamics* (Oxford University Press, London, 1938), pp. 84–85.

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ANNOUNCEMENTS AND NEWS

Book Reviews

Electromagnetic Waves and Radiating Systems. EDWARD C. JORDAN. Pp. 710+x, Figs. 284. Prentice-Hall, Inc., New York, 1950, Price $7.75.

During the past decade we have been witnessing the closing of one of the gaps in the electromagnetic spectrum. The development of the centimeter region of the spectrum has provided a common problem for engineers and physicists. To work effectively in this cooperative venture the physicists and engineers have had to learn some of each other's language. The physicists have become better acquainted with the elements of transmission lines and the electrical engineer with physical optics and field theory.

Professor Jordan has written a textbook for electrical engineers and physicists suitable for the senior and graduate years. *Electromagnetic Waves and Radiating Systems* is a valuable addition to the Prentice-Hall Electrical Engineering Series and will be used primarily by communication and electronic engineers. It is appropriate, however, that this book be reviewed for teachers of physics. The first half of the book is sufficiently general to be recommended for all students of physics whether they wish to deviate from physics toward application and engineering, or to become microwave spectroscopists, or to consider microwaves as a convenient sample of the whole spectrum for use in studying the nature of wave propagation. Even the student of physics who intends to devote his life to the wave mechanics of matter will find the modes of wave guides and antennas with their simple hand-sized patterns a graphic introduction to modes or eigenvalues of atoms.

The first quarter of the book treats field theory in a manner common to the training of upperclassmen in physics. The problems have been enlivened with modern applications to coaxial cables, free space cloth, and parallel plane transmission lines.

Since it is an engineering textbook, the rationalized mks system is employed throughout. Although most teachers of physics prefer to use the symmetrical Gaussian system in elementary field theory, a more mature student of physics should be able to use the rationalized units as well.

A chapter on guided waves provides up-to-date problems to be solved by use of Maxwell's equations subject to boundary conditions. To the student of physics who has met Maxwell's equations in a series of courses in optics, electricity and magnetism, and modern physics, and in each of them solved repeatedly the two problems of a plane wave incident upon an infinite plane conductor and an infinite plane dielectric, it will come as a pleasant surprise that there are other problems involving Maxwell's equations. The problems of guided waves are presented as sample applications of Maxwell's equations and the boundary conditions.

The concept of transmission lines is expanded to include all two-dimensional field configurations about parallel cylindrical conductors of any cross section. As a matter of good pedagogy the familiar parallel cylinders of circular cross section are treated first from the point of view of "voltages," currents and impedances, and the geometrical factors for inductance per length and capacitance per length proven to be reciprocal. Following this special case the same conclusions are drawn from field theory for the general case of all transmission lines.

Retarded potentials are derived by two methods, first by generalization from the scalar and vector potentials of the static electric and magnetic fields, and secondly, by using

the field equations to set up differential equations that the potentials must satisfy. This method of proceeding either from special to general cases or from general to special, whichever happens to be more effective, is typical of the second half of the book. The reader is impressed with the importance of considering the limitations as well as the capabilities of any method.

Jordan's approach to antennas is from the point of view of current research. Methods of making approximations are illustrated. He notes that engineering is largely a business of making good approximations. We might add also that "Physics is the fine art of making good approximations."

This is the first engineering textbook to give serious consideration to diffraction. If this textbook is typical of a trend in engineering, then any physicist performing research in the wide open field of diffraction of electromagnetic waves will do well to examine the engineering as well as the physical journals.

C. L. ANDREWS
New York State College for Teachers

Methods and Materials for Teaching General and Physical Science. JOHN S. RICHARDSON AND G. P. CAHOON. Pp. 485+vii, Figs. 547, 15.2×22.8 cm. McGraw-Hill Book Company, Inc., New York, 1951. Price $4.50.

This volume is in two parts. Part One, comprising approximately one-third of the total, presents the educational viewpoint of the authors as it relates to laboratory experiences for science teaching. Here are discussed effective ways of using apparatus and materials, demonstrations, student projects and reports, field trips, visual and auditory materials, and teaching for thinking through laboratory experiences. Also in Part One are found discussions of basic operations and devices, such as preparation of distilled water, and provisions for compressed air, electric current, high or low temperatures, and the like. The section closes with suggestions for obtaining and maintaining equipment.

Part Two makes up the remaining two-thirds of the book. In it are presented a large number of demonstrations, laboratory experiences, and projects for general and physical science, the latter including physics and chemistry. There is an appendix which supplies selected sources of demonstrations, laboratory experiences, and projects in these subjects.

The volume is one which will undoubtedly and deservedly find wide use among teachers of science in high schools and the introductory science courses in colleges and universities. It is rich in practical suggestions for interesting experiments, how to do and make things, and in ideas to supplement and vitalize conventional laboratory teaching. There is a commendable emphasis, particularly in Part One, on the importance of teaching science as a method of thinking and solving problems.

It is likely that the book will be of greatest value to teachers of physics. There are about 50 pages of experiments for chemistry, about 75 pages for general science, and some 150 pages for physics. Those described for chemistry are to a considerable extent physical in nature. Some of the experiments suggested for physics may be used also in general science and/or chemistry. However, many experiments will probably be beyond the resources and technical knowledge and skill of the average teacher in high school, particularly those who teach only general science. Many of the latter have limited knowledge experience in science and they will probably find some difficulties in following the more technical descriptions and discussions. Some of the vocabulary will be strange to many high school science teachers, especially terms relating to the use of tools, electrical and other technical equipment and processes. The meaning of such terms as "chuck," "audiofrequency transformer," "cathode-ray oscilloscope" and the like are not likely to be well-known to the average science teacher. Perhaps a glossary would have been a valuable addition to the book. Nonetheless, probably every teacher of science can find something useful in it.

Several years ago two authors from the same institution as the present authors produced a volume *Methods and Materials For Teaching The Biological Sciences* under the same publisher's imprint. This and the book under review constitute a most useful and practical pair of references for the science teacher. In them will be found a wealth of suggestions for enriching and vitalizing science courses at the high school and junior college levels. The authors and publishers are to be commended for so practical and helpful a contribution to the literature of science teaching.

VICTOR H. NOLL
Michigan State College

Fundamentals of Quantum Mechanics. ENRICO PERSICO, translated and edited by GEORGES M. TEMMER. Pp. 484. Prentice-Hall, Inc., New York, 1950. Price $6.00.

The subject of quantum mechanics is unique in the large number of new concepts that must be explained: the principles of correspondence, uncertainty, and complimentarity, the Wentzel-Kramers-Brillouin, variational and Born approximations, the nature and origin of selection rules. In addition there is a wealth of "steady-state" applications: the spectroscopy of atoms, molecules, nuclei, and solids as well as "transition" applications: the treatment of collisions of all kinds, decay, absorption, and emission.

Any author of a moderate-sized textbook must therefore limit his objectives. Enrico Persico has chosen as his primary objective a thorough explanation of the epistemological foundations of quantum mechanics, not for the expert, but for the beginning student who usually encounters difficulty at this point. His approach in Part 1 of the book is intuitive. He starts from a broad base of experimental results and provides a good historical perspective toward the climate of opinion and the contributions of various people to the early development of quantum theory. To pick an isolated example, we mention that in addition to the usual treatment of the Bohr atom, Persico illustrates the use of the Bohr correspondence principle by showing that it can be made the basis for an evaluation of the Rydberg constant.

Persico's discussion of wave mechanics is preceded by a discussion of the mathematical properties of second-order differential equations—including a simple discussion of eigenfunctions of the continuous spectrum and their

normalization. He develops the index of refraction for Schrödinger waves by comparing the least action principle of mechanics with Fermat's principle of optics. This is done in an elementary way (without the need for Hamilton-Jacobi theory). The unknown multiplying constant in the index of refraction formula is determined by the de Broglie relation between wavelength and momentum for a free particle. This is a satisfying elementary way of developing the Schrödinger equation for an electron moving in a potential field.

Persico follows his historical development (Part 1) and his wave mechanics (Part 2) by the general methods of quantum mechanics (Part 3). This order is excellent from the pedagogical viewpoint since it proceeds from the intuitive to the abstract. The third part of the book introduces the concepts of Hilbert space, operators, matrices, matrix methods, and perturbation theory.

The preoccupation of the author with the basic ideas of quantum mechanics has led to a deliberate sparsity of applications. There is little discussion of the spectroscopy of atoms, not to mention molecules or nuclei. The Thomas-Fermi, the Hartree, and the variational methods are not mentioned, and there is no discussion of collision problems of any sort.

Persico has chosen evidently, to guide the student up the main stem of the tree of knowledge with no detours among the branches. This is perhaps the best approach for the theoretically inclined student who is then able to apply the methods acquired. The average student, perhaps, needs to taste the fruit on the branches—see detailed applications—in order to appreciate the power of the tools he is acquiring.

The writer of a textbook on quantum mechanics faces in an exaggerated form the difficulty encountered by any teacher of any subject: knowledge is a many-dimensional array of facts. Any attempt to order them into a one-dimensional array, as in writing a book, or making an alphabetical index is sure to be highly arbitrary. A student will not fully appreciate matrix mechanics unless he has first learned wave mechanics—and vice versa. And he will not fully appreciate wave mechanics until he has seen detailed solutions of the Schrödinger equation.

One solution to this difficulty is to present the subject in the order: theory, application, theory, application. This has the disadvantage that the student may lose sight of the general trend of principal ideas in a welter of details. Another procedure is to present the important theoretical ideas with a minimum of details, followed by a series of applications that illustrate the principles and their interconnections. A compressed discussion of the principles of quantum mechanics aims at giving the student an immediate perspective, but some students may find it too abstract.

The conclusion we draw is that there are no royal roads to learning but many paths each suited to a particular type of student. Persico has chosen, as his title indicates, the path of developing the fundamental ideas of quantum mechanics from an elementary intuitive beginning, to their final abstract form. In this objective he has succeeded admirably.

The translation is so fluent as to be unobstrusive. Mr. Temmer has certainly contributed to the clarity of the text.

MELVIN LAX
Syracuse University

Hydrodynamics. GARRETT BIRKHOFF. Pp. 186 and xiii. Princeton University Press, Princeton, New Jersey, 1950. Price $3.50.

The appearance of a new book in the field of hydrodynamics is a rare event, and it automatically receives close attention from the all-too-small group which is seriously concerned with the subject. It is to be hoped that this book will reach a wider audience than those to whom it is of immediate technical interest because the presentation of the author is always stimulating and often challenging. Many of the topics which are considered in detail are those to which the author has made original contributions; these include free boundary theory, modeling theory, and the theory of virtual mass. The author, as a distinguished mathematician, insists on rigor in the mathematical development of the theory. His "message," however, goes beyond the special topics treated; he is deeply concerned with the logical relation between theory and experiment.

The author chooses to illustrate the connection between experiment and theory by a discussion in the first chapter of hydrodynamical paradoxes. Certainly, the notion that hydrodynamics is a subject containing its share of pitfalls is not new. Some four hundred years ago Galileo put it this way, "The movements of heavenly bodies despite their great distances from the earth have presented fewer difficulties to me than the movements of water which is within my reach." The moral, however, which Birkhoff wishes to draw from the paradoxes he presents, is that physicists and engineers minimize the importance of deductive rigor. In the words of the author, "Plausible arguments, which have often proved most fruitful in physical reasoning . . . are nevertheless fallible. Among these are the arguments that 'small causes produce small effects' and 'symmetric causes produce symmetric effects.'"

The reader will find in the second chapter an excellent presentation of free boundary theory including the recent developments in this field. For a long time free streamline flows were considered to be mathematical curiosities, but the past ten years has brought a lively interest with many important advances in the theory. The author remarks that this subject, which received important contributions from physicists of the nineteenth century, owes its present progress to mathematicians and engineers. He calls for renewed interest on the part of the physicist, and it may very well be that his discussion of the subject will stimulate the contributions which the physicist can certainly make.

The third chapter considers modeling and dimensional analysis. While the physicist finds himself on familiar ground here, he will be forcibly reminded of the preoccupation of the engineer with the modeling technique. The engineer, however, may be baffled rather than edified by the discussion. It is, perhaps, the physicist's turn to be baffled in the fourth and fifth chapters which are titled "Group Theory and Fluid Mechanics" and "Virtual Mass and Group Theory." The author, however, has a serious

point which consists in the demonstration that group theory brings out "a hidden order" in fluid mechanics. The development of these applications of group theory has not been published before.

Throughout the book, the author lays about freely, cudgeling the engineer and physicist with impartiality. He repeatedly attacks the use by the engineer or physicist of "plausible arguments" or appeals to "common sense." In defense of the physicist let it be said that he has more interest in basic concepts than in mathematical rigor. One may support this view by pointing to the physical concept of the boundary layer introduced by Prandtl. This relatively recent physical notion made possible impressive and rapid advancements in fluid mechanics. The physicist will freely grant a central role to logic and rigor in the development of a theory; but, as he reads the subtitle of this book "A Study in Logic, Fact, and Similitude," he will seize upon the word "fact" and ask about the physical concepts to which the facts lead and through which they are integrated.

It should be clear that the physicist will be stimulated by this book, perhaps even to a serious consideration of some of the problems in fluid mechanics. If so, the book has gone far toward accomplishment of one of its aims.

MILTON S. PLESSET
California Institute of Technology

Advances in Radiochemistry. E. BRODA. Pp. 152+xi. Cambridge University Press, London and New York, 1950. Price $2.75.

This monograph is devoted to a treatment of advances in radiochemistry since the publication of Paneth's *Radioelements and Indicators* and Hahn's *Applied Radiochemistry.* Radiochemistry is defined by Broda as the chemistry of bodies which are detected through their nuclear radiations; thus both applied radiochemistry (the use of nuclides as tools in chemistry) and radiation chemistry (the chemical effects of radiation) are excluded from his book.

Methods of preparing nuclides and separating them comprise the bulk of the monograph. The use of carriers, radiocolloids, partition between solvents, volatilization, and electrodeposition are reviewed. Considerable space is devoted to a thorough discussion of the physics and chemistry of the production of radioelements by neutron irradiation and of nuclear fission. The production of radioisotopes of elements 43, 61, 85, 87, 93, 94, 95, 96, as well as tritium and radiocarbon, is discussed. A useful chapter reviews recent work on the Szilard-Chalmers effect, recoil collection, and aggregate recoil. Coverage of the newer instruments, technics of measurement, and of the role of chemical differences between isotopes are too incomplete to be of much value.

Few experimental data are included and results are not critically evaluated; in many cases no effort is made to point out difficulties and uncertainties in the experimental work or inadequacies in theory. This weakness along with a general failure to discuss results in terms of underlying principles, makes the book unsuitable as a text. It should serve as a very helpful comprehensive organized bibliography for workers in this and related fields, and its value for this purpose is greatly enhanced by inclusion of references to much of the classical work with the naturally occurring radioelements. An authoritative modern treatise on this important subject is, however, still needed.

MAX T. ROGERS
Michigan State College

Economic Aspects of Atomic Power, An Exploratory Study under the Direction of SAM H. SCHURR and JACOB MARSCHAK. Pp. 289+xxvi, Figs. 6, Tables 32, 17½×26 cm. Princeton University Press, Princeton, New Jersey, for the Cowles Commission for Research in Economics, 1950. Price $6.00.

Seldom has the social value of competent economic analysis been demonstrated so clearly as in the work under review. The conclusions reached should prove of great importance both to the public and its representatives and to the managers of private industry in the formulation of long-range plans. The method of analysis employed in the investigation holds an interest of its own for professional economists. Less obvious but not less significant is the fact, which may be readily inferred from Marschak's account in the preface of the background of the investigation, that fruitful results from economic research, as in technological research, can be won only with adequate resources. The day has passed, if it was ever here, when important questions could be settled directly from general principles.

Predicting the economic effects of a body of scientific and technical knowledge which is itself rapidly changing is a hazardous task. The authors have minimized the difficulties involved by examining only those potential uses of atomic power which loom large on the horizon and by enumerating with precision the assumptions made at each phase of the analysis. The end-product is a series of forecasts based on the assumptions most reasonable in the light of available evidence, with suggestions as to possible alternative results when and if future developments invalidate the assumptions made.

The central conclusions of the investigation are that atomic power: (1) will probably result in an American national income about one percent higher in the next two or three decades than that which would have obtained under traditional power techniques; and (2) will probably prove of appreciably greater value in the same period to less industrialized nations than to the United States. The reasons for the second conclusion are perhaps more readily grasped than are those for the first. Some nations which have historically been by-passed by the tide of industrialization have lacked the necessary water power or coal resources. The advent of the new, nearly costless fuel removes this obstacle. Secondly, a nation, such as ours, with heavy investments sunk in older power production facilities will often find the prospective outlays for producing electricity by traditional means to be less than would be required to produce it from atomic power. While the old facilities and skills abound, the ultimate economic potentialities of the new technique will not (and should not) be fully tapped (unless subsidized as part of the armament program). Less advanced countries need not defer use of atomic power pending the attrition of existing investment

and the corresponding rise in the incremental costs of old style power production. While such considerations suggest a shift in the balance of international economic power to the disadvantage of the United States, the ability of the rest of the world to make use of atomic power will be at least partly influenced by the willingness of this country to provide others with the equipment and know-how necessary to produce and consume atomic power economically.

Although the anticipated aggregate effects of atomic power development in America do not promise to be large for the next generation or two, individual industries which are, or with appropriate prices may become, important consumers of electricity may be seriously affected. These industrial effects will vary in their impact upon the different regions of the country, just as will the international effects of atomic power. The aluminum industry will be enabled to continue its expansion freed from the necessity of locating at cheap hydroelectric outlets, and capable of locating closer to either the raw material or the market. Similarly, the production of phosphate fertilizer could be relocated near the phosphate rock deposits of Florida and the traditional sulfuric acid method of production abandoned for the electric process. The cement industry may eventually be able to use hot gas from atomic reactors, but not electricity. In the iron and steel industry, the tendency of steel producers to locate near markets and scrap supplies will probably be accelerated; if the experimental method of producing sponge iron from ore and hydrogen develops, atomic electricity may be employed to produce hydrogen by electrolysis of water, and iron reduction could move close to the iron mines. If the latter developments materialize in the iron and steel industry, its locational pattern will be revolutionized and the existing advantages of large scale operation will probably be sharply diminished if not eliminated.

Of the industries analyzed, the above offer the most promising markets for atomic electricity. Yet, even in these, the cost-savings anticipated are more or less contingent upon the development of production techniques which an uncertain future may or may not yield up. On the other hand, as the authors themselves emphasize, the ultimate consequences of the new science may prove vastly greater than can be predicted from its present visible potentialities. Competent analysts can forecast the economic consequences of existing knowledge. No one can tell what knowledge is yet to come.

JACOB SCHMOOKLER
Michigan State College

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. 19, 434 (1951)].

Cronin, James Watson (*J*), 3825 Hanover St., Dallas, Texas
Faris, John Jay, 1710 A St., Forest Grove, Ore.
Garrity, John Lawrence, 92 Covington St., Springfield, Mass.
Ginn, James Lawrence (*J*), 133 Lincoln St., Liberty, Mo.
Grismore, Roger, 1914 Day St., Ann Arbor, Mich.
Haughney, Louis C., 1005 Brown Ave., Erie, Pa.
Hyslop, William Henry, University of Denver, Denver 10, Col.
Kline, Raymond Milton (*J*), 223 Welch St., Ames, Iowa
Lowe, Forrest Gilbert, 519 E. 4th St., Maryville, Mo.
Lucian, Anthony D., 1714 Green Rd., Cleveland 21, Ohio
Lyman, Arthur Lee, Jr. (*J*), 52 South 8th Ave., Mt. Vernon, N. Y.
Maupin, Robert Talmadge (*J*), 920 W. Chestnut St., Louisville 3, Ky.
McLeroy, Edward Glenn, Jr. (*J*), 517 Jefferson Ave., East Point, Ga.
Miller, Charles G., Department of Physics, University of California, Santa Barbara, Calif.
Nelson, Iral Clair (*J*), 1410 West 4th St., Eugene, Ore.
Pedrotti, Leno Stephano (*J*), Dept. of Physics USAFIT Wright Patterson AFB, Dayton, Ohio
Penza, Silvio Salvadore (*J*), 643 Pleasant Valley Parkway, Providence, R. I.
Robinson, Daniel L. (*J*), 201 West 77th St., New York 24, N. Y.
Schweizer, Leslie Robert, Box 267, Blacksburg, Va.
Shacklett, Robert Lee, 1024 Buckingham, Fresno, Calif.
Taylor, Charlie Tilphord, Jr., 300 N. Washington St., Albuquerque, N. M.
Vellequette, Murlin Joseph (*J*), 4327 Bales Rd., Toledo 13, Ohio
Vigue, Kenneth James, 5210 T St., SE, Washington 19, D. C.

AMERICAN JOURNAL OF PHYSICS VOLUME 19, NUMBER 8 NOVEMBER, 1951

RECENT MEETINGS

Western Pennsylvania Section

The Western Pennsylvania Section of the American Association of Physics Teachers met at Geneva College, Beaver Falls, Pennsylvania on May 5, 1951. Thirty-seven members and guests were present. PROFESSOR CHARLES WILLIAMSON, *Carnegie Institute of Technology*, acted as chairman. After greetings by DR. C. M. LEE, president of *Geneva College*, the following papers were presented:

The synthesis of atoms and their isotopes. ROBERT C. COLWELL, *West Virginia University*.—A satisfactory theory

of atomic synthesis must explain: (1) The very great preponderance of the even-numbered elements over the odd-numbered elements; (2) the existence of two isotopes only for odd-numbered elements; (3) the absence of elements heavier than uranium. This is done by supposing that in each star the original particles—electrons, protons, neutrons, and alpha-particles—were stratified according to their weights. Then the increasing pressure as the star contracted produced all the elements of the periodic table with their isotopes. The general formula was given about thirty years ago by Harkins. In modern symbols each element would be given by n_2He^4 for the even-numbered elements and $n_2He^4 - {}_1H^3$ for the odd-numbered elements.

An acoustic interferometer. K. J. METZGAR, *University of Pittsburgh.*—An ultrasonic interferometer has been developed for use in a sophomore atomic physics laboratory. The purpose of the instrument is to enable the student to measure directly the wavelength of sound in various gases and from this compute the velocity of sound in each gas, the number of degrees of freedom of the gas molecules, and the classical specific heat at constant volume. The apparatus consists of a piezoelectric quartz crystal mounted in the end of a chamber opposite a reflector on a micrometer screw. The quartz crystal is driven at its resonant frequency of one megacycle per second by a simple oscillator. The variation in crystal current as a function of reflector distance is detected by a thermocouple and galvanometer. The separation between nodes in the standing wave can be obtained easily from the resulting curve.

Projects in electricity laboratory. CHARLES WILLIAMSON, *Carnegie Institute of Technology.*—After performing ten standard experiments, the student chooses a project from a list of ten for which the department has the necessary apparatus. Some of these projects require independent thinking and planning; others require only intelligent and selective use of printed references. Three men ordinarily work together, planning and executing their project in four weeks, with minimum help from the instructor. Each team makes oral reports to the class, and hands in a written final report. Surprising enthusiasm is generated, and much is learned about the management of a small research project. A list of project titles appears in this paper.

An anomaly in the determination of the coefficient of kinetic friction. EDWARD I. RUBENDALL, *Thiel College.*—In a laboratory experiment the student is given a 4 in. ×4 in. ×9 in. wooden block having two tapered edges, a piece of brown wrapping paper, string, a pulley, a hanger, and laboratory masses. He is asked to determine for himself the facts of kinetic friction such as dependence on surfaces in contact, independence of area of contact, independence of velocity, and proportionality to normal force. Additional mass is obtained by placing laboratory masses on the block. The data indicate that the expected proportionality does not exist. On plotting normal force as abscissa and frictional force as ordinate the slopes of the lines are the same, but extrapolation indicates an intercept on the ordinate axis. The slope is taken as the coefficient of kinetic friction and the intercept as a constant frictional force caused by the torque exerted by the frictional force about the center of mass.

Demonstrations of nuclear ray tracks. CECIL O. RIGGS, *Waynesburg College.*—Exhibits of Eastman NTB plates which has been soaked in thorium nitrate solution and of NTA plates which had been exposed to neutrons were set up with magnifications of about 1000 and 400. Difficulties and precautions in the use of the films were mentioned. Soaking the films in about 30-percent glycerine was recommended to prevent cracking and peeling. Requirements of the microscope to be used were briefly considered. Satisfactory measurements cannot be made if movement of the slow motion screw moves the field; binding the microscope against the ways with rubber bands was suggested to avoid this.

Effective recitations. O. H. BLACKWOOD, *University of Pittsburgh.*

Demonstrations with simple equipment. GORDON M. DUNNING, *Indiana State Teachers' College.*—To help prepare physics teachers for the secondary schools, a series of demonstrations using simple and inexpensive equipment has been developed at the State Teachers College, Indiana, Pennsylvania. Six of these demonstrations were presented at the meeting: chain reaction using matches, a mass spectrograph using a remade pin-ball machine, the collisions of alpha-particles in a Wilson cloud chamber, the mixing of colors, and two demonstrations on Archimedes' principle.

A seminar for physics majors. W. C. KELLY, *University of Pittsburgh.*—A student section of the American Institute of Physics has been organized at the University of Pittsburgh, and its activities combined with those of a long established seminar for undergraduate physics students. At weekly meetings, students hear papers on current topics in physics, its history, and its philosophy. Most of the papers are presented by students; a few, by staff members and visitors, Members of the section receive *Physics Today.* The section has been a profitable activity, giving students: (1) experience in addressing a meeting, (2) a sense of preparing for a well-organized profession, and (3) information about both the background of physics and its present activities.

At the business meeting, the secretary announced that DR. R. E. WARREN, president of the section, had been recalled to active duty in the Army and had been forced to resign his office as of January 1, 1951. The Section thereupon elected Professor CHARLES WILLIAMSON as president pro tempore to fill the unexpired term. Professor GORDON M. DUNNING, *State Teachers College,* Indiana, Pennsylvania, was elected vice-president. The committee on high school certification requirements, G. M. Dunning, chairman, presented its report, reviewing steps that have been taken by various groups to raise the certification requirements for teaching science in Pennsylvania. The section passed a resolution endorsing, in slightly amended form, the recommendations of the Pennsylvania Academy of Science, con-

cerning teacher certification, as presented at the meeting of the Academy on April 7 and 8, 1950. A proposal was presented to change formally the name of the organization from "Association of Physics Teachers of Western Pennsylvania and Environs" to "Western Pennsylvania Section of the American Association of Physics Teachers." Final action on this proposal will be taken at the fall meeting. A vote of thanks was given to DR. LESLIE FALLON, *Geneva College*, for making arrangements for this meeting.

The fall meeting of 1951 will be held at Carnegie Institute of Technology on the first Saturday in December, and the spring meeting of 1952 at Allegheny College on the first Saturday in May.

W. C. KELLY, *Secretary*

Colorado–Wyoming Section

The annual meeting of the Colorado-Wyoming Section of the American Association of Physics Teachers was held April 27 and 28, 1951, at the University of Denver. As has been the custom, the meeting was held in conjunction with the Colorado-Wyoming Academy of Science.

The following program of papers was presented in Science Hall on the University Campus:

Colorimetry and its applications (Invited paper). H. M. SULLIVAN, *Central Scientific Company.*

Large-sized apparatus in lecture demonstrations in physics. W. H. KADESCH, *Adams State College.*

The Stroude and Oates induction bridge. W. H. HYSLOP, *University of Denver.*

Analogies as an aid to thought. PAUL F. BARTUNEK, *Colorado School of Mines.*

Some thoughts on teaching physics in technical schools. PAUL F. BARTUNEK, *Colorado School of Mines.*

At the business meeting DR. PAUL E. BOUCHER, *Colorado College*, was elected chairman for the following year. DR. W. B. PIETENPOL, *University of Colorado*, was re-elected to the Executive Committee of the American Association of Physics Teachers.

ROBERT O. BOCK, *Chairman*

Oregon Section

The 57th meeting of the Oregon Section of the Association was held at Reed College, Portland, on May 19, 1951. After the meeting opened, but before the program got underway, a lively exchange of ideas took place on the relationship, actual or desired, between high school physics and high school physics teachers on the one hand, and the members and activities of the AAPT on the other.

The following program was presented:

X-ray spectrometers. KENNETH E. DAVIS, *Reed College.*

The near-neighbor effect in cosmic-ray stars. KENNETH M. KING, *Reed College.*

The angular distribution of secondary electrons from platinum. JOHN J. FARIS, *Pacific University.*

The detection of soft x-rays with a scintillation counter. LYMAN A. WEBB, RONALD S. PAUL, and FRANCIS E. DART, *University of Oregon.*

Energy loss of beta-rays in solid anthracene. RONALD S. PAUL, *University of Oregon.*

The systematics of alpha-decay. E. G. EBBIGHAUSEN, *University of Oregon.*

The design and construction of a solid boron neutron detector. DALE M. HOLM, *Oregon State College.*

Depolarization in scattering of thermal neutrons. DAVID B. NICODEMUS, *Oregon State College.*

A report was brought from the executive committee meeting, held in New York on February 1, by WILL V. NORRIS, *University of Oregon*, representative of the Oregon Section on the executive committee. The place of meeting for the autumn gathering of the Section was determined by accepting the invitation extended by the University of Washington.

At the business meeting, the following officers were elected for the coming year: *President*, WILLIAM L. PARKER, *Reed College; Representative* on the Executive Committee of AAPT, WILL V. NORRIS, *University of Oregon; Historian*, B. GODFREY, *University of Portland; Secretary*, FRED W. DECKER, *Oregon State College.*

FRED W. DECKER, *Secretary*

. . . there was nothing more interesting to watch than the progress of science. Its progress was exceedingly gradual. It was like the progress of man going through a swamp, with islands of firm earth in between. The advances would be very slow from week to week, but at the end of a year it was very great, and at the end of ten years it was enormous.—RUTHERFORD, 1909.

Proceedings of the American Association of Physics Teachers

The East Lansing Meeting, June 26–28, 1951

THE June meeting of the American Association of Physics Teachers, held jointly with the American Society of Engineering Education at East Lansing, Michigan, will be remembered as a notable addition to the series inaugurated by the Association in 1937. Summer meetings have a vacation flavor and are approached with such a fine spirit of friendliness and good will that success is inevitable. Such was the case at East Lansing. Pleasant surroundings, reasonable weather, and a well-diversified program planned by PRESIDENT-ELECT WEBB, and his associates, made a combination both stimulating and enjoyable.

The first formal meeting was held Tuesday afternoon and was devoted to a symposium on the "Teaching of Fluid Dynamics." This was introduced in a very able manner by DR. SEEGER. It was apparent that fluid dynamics is an almost neglected topic in the ordinary physics program at both the elementary and intermediate levels, while much of the more advanced theoretical work is either incorrect or does not apply. Much can and should be done to rectify the situation, and Dr. Seeger should be congratulated for his lucid presentation of the opportunities and responsibilities of the teachers in this field. Adding greatly to the program were talks by DR. GRIFFITH on the "Shock Tube and Its Functions" and PROFESSOR SUTTON on "Interesting Aspects of Teaching Elementary Fluid Dynamics." In connection with this program, lecture demonstration apparatus, valuable in teaching fluid dynamics, was on exhibit in the laboratory. This apparatus included well-designed wind generators and appropriate equipment for demonstration and measurement on a lecture scale of all the major phenomena in fluid dynamics.

Following this session two McGraw-Hill films, "Uniform Circular Motion" and the "Otto Cycle" were shown. These largely result from the efforts of the AAPT Committee on Visual Aids and, in the words of the Chairman, PROFESSOR ZEMANSKY, the committee should have the praise and he himself all the blame.

The popularity and success of television prompted the evening program. Meeting in the television auditorium, we were favored with a well-planned demonstration lecture by PROFESSOR BASCOM of Wayne University which was telecast for the audience. The possibilities of the use of this medium for science education was made apparent to everyone.

Wednesday morning was given over to a general session with the A.S.E.E. An address of welcome by PRESIDENT JOHN A. HANNAH, and a talk on engineering education by DEAN DAWSON of Iowa State University will be particularly remembered.

A symposium on "Ultrasonics" was arranged for Wednesday afternoon. PROFESSOR HEIDEMANN, a recognized authority in this field, introduced the subject with an extensively illustrated talk showing the many ways in which ultrasonic fields can be made visible. Of major interest were the optical methods for which he has been responsible and which are capable of giving precise, quantitative information as to the nature of the fields. In natural sequence there followed a talk by DR. JULIAN FREDERICK on applications of ultrasonics using pulsing techniques, and a discussion of power measurements in ultrasonics by DR. OSKAR MATTIAT. The session was concluded with the showing of a film, "Ultrasonic Effects" (courtesy of the Brush Development Company).

The evening banquet will be remembered as one of the high lights of the meeting. Appropriate introductions by Toastmaster Zemansky, a worthwhile and spicy address entitled "Solomon's House" by T. A. BOYD of General Motors Research, and NEWTON GAINES and his guitar were the features. This last feature deserves special mention. Mr. Gaines sang a series of (scientific?) ballads, the last, entitled "Thermodynamics" and dedicated to the Toastmaster, so captivated the audience that it may be worth recording here for the entertainment of all.

Thermodynamics

(*Known as 'Kirk's Lament' in the University of Wisconsin, where it probably originated.*)*

Tune: "Battle Hymn of the Republic."

* Dr. Langer of the University of Wisconsin gave this song to Mr. Gaines several years ago. A private communication to the editor from Dr. Langer states that he has no recollection as to how he first obtained the song or who might be responsible for it.

Free energy and entropy were whirling in his brain
With partial differentials and Greek letters in their train,
For sigma, gamma, theta, delta, epsilon, and pi
Were driving him distracted as they turned before his eye.

Chorus:

Glory, glory, dear old thermo
Glory, glory, dear old thermo
Glory, glory, dear old thermo
We'll pass you by and by.

Heat content and fugacity they danced within his mind
Like molecules and atoms that you never have to wind,
With logarithmic functions jitterbugging in his dream
And partial molar quantities in saturated steam.

Chorus.

They asked him on the final if a mole of any gas
In a vessel with a membrane through which hydrogen could pass,
Were compressed to half its volume what the entropy would be
If two-thirds of delta sigma equaled half of delta *P*.

Chorus.

He said he guessed the entropy would have to equal four
Unless the second law might bring it up a couple more
But then it might be seven if the Carnot law applied,
Or it might be almost zero if the delta *T* should slide.

Chorus.

The professor read his paper with a corrugated brow
For he knew he'd have to grade it but he couldn't figure how,
Until an inspiration on his cerebellum smote
Ane he seized his trusty fountain pen and this is what he wrote:—

Omit Chorus

"Just as you guessed the entropy, I'll have to guess your grade,
But the second law won't raise it to the mark you might have made,
For you might have made a hundred if your guesses had been good—
But they weren't, so you'll repeat this course until it's understood!"

Chorus.

Thursday morning was devoted to contributed short papers, thirteen in number, enlivened by many questions and comments from the floor. The tone of the meeting was set by our genial chairman, R. F. PATON, and his dictatorial attempts to limit the time of the speakers and the discussion periods. In fact, after his first emphatic remarks on these subjects every clock in the building stopped, only to resume operation during presentation of the final paper. The session closed on schedule but participants still had the last word for PROFESSOR GAINES' apparatus for demonstrating dissonance (no reference to R.F.P.), and PROFESSOR BARBOUR's diffusion cloud chamber, as well as spirited blackboard arguments *re*: vectors, quite consumed the lunch period for some people.

The concluding session Thursday afternoon was devoted to nuclear engineering and associated problems. A group from Oak Ridge, J. H. FRYE, JR., J. A. SWARTOUT, and R. N. LYON, gave papers on metallurgical, chemical, and heat transmission problems, respectively, which must be solved before nuclear power can be realized. Although much of this was necessarily presented in a very general fashion everyone was impressed by the formidable physical problems which are being faced and solved to achieve this end.

Being a local member, I should not say much about the hospitality of the Michigan State group. However, I think the social needs were adequately provided for. The A.S.E.E. planned an extensive program for the ladies. This included campus and off campus tours, special lectures, and parties. All were entertained at tea at the home of President and Mrs. John A. Hannah.

C. D. HAUSE
Michigan State College

Invited Papers and Reports

Papers and Special Events

The education, employment, and earnings of physicists. MARSH WHITE, *The Pennsylvania State College*, M. H. TRYTTEN, *Office of Scientific Personnel*, AND ROBERT W. CAIN, *Bureau of Labor Statistics, U. S. Department of Labor.*

Science on television. A demonstration. EDWARD R. BASCOM, *Wayne University.*

The WFIL-TV University of the Air. ARMAND L. HUNTER, *Temple University and Michigan State College.*

General Session

(Sponsored by the Engineering College Research Council)

Presiding: G. A. ROSSELOT, *Vice-President* ASEE, *Georgia Institute of Technology.*

Research and development: The National balance sheet in June 1951. E. A. WALKER, *Research and Development Board.*

Research and Development: The nation's potential in educational institutions. A. F. SPILHAUS, *University of Minnesota.*

National Science Foundation developments. A. A. POTTER, *National Science Foundation.*

Symposium on the Teaching of Fluid Dynamics

(Sponsored by the Division of Fluid Dynamics of the American Physical Society)

Presiding: EGON A. HIEDEMANN, *Michigan State College.*

Fluid dynamics in physics teaching. R. J. SEEGER, *Naval Ordnance Laboratory and the Institute of Applied Mathematics, University of Maryland.*

The shock tube and its functions. WALTER BLEAKNEY AND WAYLAND C. GRIFFITH, *Princeton University.*

Interesting aspects of teaching elementary fluid dynamics. R. M. SUTTON, *Haverford College.*

General Session—with ASEE

Presiding: HENRY H. ARMSBY, *Vice-President* ASEE, *U. S. Office of Education.*

Address of welcome. J. A. HANNAH, *President, Michigan State College.*

Engineering education, a bridge between ignorance and understanding. F. M. DAWSON, *President ASEE, University of Iowa.*

Panel discussion of committee on improvement of teaching. L. E. GRINTER, *Illinois Institute of Technology,* H. P. HAMMOND, *The Pennsylvania State College,* A. P. COLBURN, *University of Delaware,* AND J. F. CALVERT, *Northwestern University.*

Symposium on Ultrasonics

Presiding: THOMAS H. OSGOOD, *Michigan State College.*

Methods of making ultrasonic fields visible. EGON A. HIEDEMANN, *Michigan State College.*

Applications of ultrasonic pulse techniques in research and testing. JULIAN FREDERICK, *University of Michigan.*

Power measurements in ultrasonics. OSKAR MATTIAT, *Hypersonics Laboratory, Brush Development Company.*

Ultrasonic effects-film. *Brush Development Company.*

Some Technical Problems Associated with the Realization of Nuclear Power

Presiding: G. P. BREWINGTON, *Lawrence Institute of Technology.*

Metallurgical problems in nuclear industry. JOHN H. FRYE, JR., *Metallurgy Division, Oak Ridge National Laboratory.*

Some chemical research problems in the development of nuclear reactors. J. A. SWARTOUT, *Oak Ridge National Laboratory.*

Heat transmission problems. RICHARD N. LYON, *Reactor Technology Division, Oak Ridge National Laboratory.*

Contributed Papers, with Abstracts

A single session presided over by R. F. PATON, Secretary of the Association, was devoted to presentation of the following contributed papers:

1. Projects in electricity laboratory. CHARLES WILLIAMSON, *Carnegie Institute of Technology.*—After performing nine or ten standard experiments, the student chooses a project from a list of nine or ten furnished by the department. Some of these are novel, requiring independent thought; others are routine, requiring only intelligent use of reference books. Three men ordinarily work together, planning and executing the project in four weeks, with minimum help from the instructor. Each team makes oral progress reports to the class, and hands in a written final report. Surprising enthusiasm is generated, and much is learned about the management of a small research project. A list of project titles was given in this paper.

2. An improved apparatus for demonstrating an oscillatory discharge. EDWIN S. FOX, *Western Michigan College of Education.*—An oscillatory circuit having a frequency range of from 400 to 4000 cps is so arranged that its condenser is automatically charged many times each second by a pulse from a thyratron. In the interval between pulses an oscillatory discharge takes place; this was made visible by means of a Dumont Model 304 oscilloscope.

3. A physical model to demonstrate nuclear and paramagnetic resonance. E. F. CARR AND C. KIKUCHI, *Michigan State College.*—We have attempted to illustrate with a physical model some of the basic principles in the theory of nuclear and paramagnetic resonance absorption. This model was constructed to demonstrate the effects of space quantization of the electron and the nuclear spin, and also to make clear the various factors that contribute to the total magnetic field at the position of either the electron or the proton. We have also attempted to show what might happen when a paramagnetic ion absorbs a photon.

4. Motor-driven vibrator units for the measurement of capacitance. D. S. AINSLIE, *University of Toronto.*—This paper described two adaptations of the Amglo battery-driven motor, used to measure capacitance. In the first adaptation, the motor was operated on an ac circuit leaving the reed contactor, originally employed as the control, free to charge and discharge a condenser. Two of these units were employed in a modification of a Kelvin bridge which gave excellent results with a minimum of standard resistors. A second adaptation was devised for operation from a 6-volt battery which gave a frequency of charge and discharge of approximately 60 cycles per second.

5. Two lecture demonstrations. W. W. SLEATOR, *University of Michigan.—1. Center of percussion.*—This experiment demonstrated that the center of oscillation of a disk pendulum is also the center of percussion. An iron cylinder had a notch around it at the middle, depth just $\frac{1}{4}R$. The cylinder lay on a light board supported on two

rollers on a level table, with the axes of cylinder and rollers parallel. When the cylinder was set rolling by a cord in the notch, and pulled off above the center, the board did not move. *2. Longitudinal waves.*—An old wave machine intended to show transverse waves was used by projection to show longitudinal waves also. It demonstrated that a particle in the midst of a condensation has its maximum velocity. A set of wood disks were equally spaced along a horizontal rod. When the end disk was pushed along toward the others it gathered them into a solid moving column, and it became obvious that

(velocity of material)/(velocity of waves)
$= -$ (change in volume)/(original volume) $= -$ strain.

It was shown that this equation holds in condensation and rarefaction, and for velocity of waves away from the origin and toward the origin.

6. Lenses for laboratory-built spectrographs. RALPH A. LORING, *University of Louisville.*—Several spectrographs have been built for use in the modern atomic physics laboratory particularly for photographing sodium-like spectra. Various lenses have been tried,—spectrometer telescope lenses, camera lenses, war-surplus lenses. The final choice was the simple lens corrected for spherical aberration.

7. Pressure energy and Bernoulli's principle. GEORGE LINDSAY, *University of Michigan.*—Through an examination of some thirty textbooks of physics it appears that more than one-half of them teach that when an incompressible liquid is subjected to a pressure p, the liquid has a potential energy due to the pressure alone, of p units of energy per unit volume, in addition to any gravitational potential energy or kinetic energy it may have. In conformity with this viewpoint, Bernoulli's equation

$$p+\tfrac{1}{2}\rho v^2+\rho gh=C$$

for a fluid in a steady state of flow, is held to be an expression of conservation of energy per unit volume of the fluid, as it moves along a stream line. It is shown that the arguments given for the existence of this pressure energy are invalid, lead to violation of the work-energy principle of mechanics, and to a contradiction of the conservation of energy principle itself. The conclusion is that there is no basis for the assertion of pressure energy in an incompressible fluid.

8. A study of subjects for motion pictures for teaching physics. ROBERT PETRY, *University of the South.*—Certain subjects in physics have been suggested by AAPT members for treatment by motion pictures; rough preliminary animations of some of these and of similar topics have been made up, to try to estimate their suitability for permanent films. The first two deal with SHM of the pendulum and with Lissajous figures of 1/1 ratio and of different phase relations. The third, on the Commutator, uses both animations and models to test out the subject. The use of color merely to set off different parts of drawings from each other was tried, as applied to transverse standing waves, longitudinal standing waves, Huygens' principle and double wave fronts in doubly-refracting crystals, and diffraction by wavelets from two slits. In contrast, color has been used to indicate difference of color in interference by light from two slits. The films give indications of the merits and weak points of these topics as subjects for animations. (Given by title.)

9. Study of sliding friction. CHARLES A. MANEY, *Defiance, Ohio.*—Between limits, the force applied to produce motion of a body over a nonlubricated surface does not produce acceleration, but is absorbed by friction. The coefficient of sliding friction is not a constant, but for all materials tested, both metals and nonmetals, varies directly with the logarithm of the speed, for medium speeds. In the case of lead on steel, from speeds of 1 cm/sec to 21 cm/sec, the coefficient is found to increase by 76 percent, —from 0.165 to 0.290. The range of variation of the coefficient with respect to equal velocity changes differs widely for different materials. At least in the case of wood surfaces the coefficient of friction decreases as the normal force increases. In the case of fibrous surfaces as wood or leather, the coefficient decreases with repeated performance. In the case of metal surfaces the coefficient depends upon the degree of "cleansing" of the surfaces, more particularly upon the degree to which the product of the abrasive action of friction is removed. This was illustrated in the case of a brass surface sliding over a steel surface.

10. Graduate level laboratory training for government research scientists. GEORGE ABRAHAM,* *Naval Research Laboratory.*—Specialized laboratory training in physics and electrical engineering at the graduate level has served to bridge the gap between these fields for research scientists. Specific research tools have been developed in this program which has benefited both the government research laboratory and the cooperating university. Specific examples are discussed and the nature and conduct of the program are described.

* Also Lecturer (part time) in physics and electrical engineering, *University of Maryland.*

11. Experimental derivation of the Helmholtz dissonance curve using modern electrical apparatus. NEWTON GAINES, *Texas Christian University.*—For a number of years the "two-violin" dissonance experiment, performed by the class as a whole, has formed the keystone of the acoustics course required of all our music majors. The fixed tone, produced by a reed organ blown by centrifugal pump, is blended with the reedlike tone of variable pitch produced by a combination of audio-oscillator, overdriven amplifier, and loudspeaker. By ratios of integers that represent harmonics, we predict each interval, using the main division lines of scales A and B of a large demonstration slide rule to represent the harmonic structure, respectively, of fixed and sliding tones. All frequencies are measured by Stroboconn; degrees of dissonance are estimated by simple formula and checked by ear. Pentatonic and the pure major and minor diatonic scales are shown to follow.

12. Fraunhofer multiple-slit diffraction patterns with infinite sources. A. E. SMITH AND C. D. HAUSE. *Michigan State College.*—Fraunhofer double-slit diffraction patterns and their dependence upon source dimensions are commonly treated in textbooks and lend themselves well to experimental verification in the laboratory. The experiment usually involves a laboratory model of Michelson's stellar interferometer. The possibility of obtaining an effective increase in resolution by using multiple slits within the same aperture space and their associated secondary maxima in the patterns was investigated. General expressions for the intensity distribution for the N-slit pattern with finite sources both in the form of a uniformly radiating slit and a disk were obtained. Using characteristics of the secondary maxima an effective increase in resolution of $1/N$, $N>2$, was indicated. This was verified experimentally.

13. Study tips for the physics student. JAMES G. POTTER, *A. & M. College of Texas.*—Some specific suggestions of practices which have been found helpful by students in gaining a working mastery of physical theory were presented. A technique of self-questioning is developed, including questions such as "Is this fact new to me? If not, did I already know it as a result of critical observation or as a result of prejudice? If the fact is new, is it something I should have expected? Why did I not recognize it before? Why did I not appreciate the importance of obtaining such a fact? Did I already have this concept? Is the explanation I have received equivalent to the one I would have given? Is the concept the sort I probably would have used if I had pursued the subject far enough on my own? If I had encountered this new term without its definition, how would I have defined it? Would I have arrived at the same law if I had endeavored to reason from the same facts and concepts? Why is the law contrary to my expectation? Why do the authorities arrive at a result different from mine? Does this step in my solution of this problem do exactly what a physical principle or law says is correct and assume nothing further?" Criteria for the timing of systematic reviews were discussed.

14. Vector quantities in introductory physics. J. G. WINANS, *University of Wisconsin.*—When physical quantities are defined in terms of length, time, and force instead of length, time, and mass, definitions consist of combinations of two vectors and one scalar instead of two scalars and one vector. Definitions of quantities like work, area, torque, volume, pressure, mass, stress, surface tension, and rotational inertia contain multiplication or division of vector quantities. Definitions involving multiplication of vectors apply for vectors in any direction. Definitions involving division of vectors are unique only for parallel vectors or perpendicular vectors. Pressure defined by $p=\mathbf{F}/\mathbf{A}$ has definite meaning only when **F** and **A** are parallel. Since $\mathbf{F}=p\mathbf{A}$, pressure is a scalar with no direction. Intensity $\mathbf{I}=P/\mathbf{A}$ behaves as a vector in the direction of **A**. Surface tension, $\mathbf{S.T.}=\mathbf{F}/\mathbf{d}$ with **F** perpendicular to **d** behaves as a vector perpendicular to both **F** and **I**. A quantity is a vector if it has magnitude and direction and adds by the vector method. Of several quantities which fit this specification, such as force or effective alternating current, one cannot be considered as a more real vector than the other.

Effective deployment means that those who engage in military projects should do so in the most effective way possible with regard not only to solving immediate problems, but also to increasing their ability to participate effectively in whatever situation may arise in the future. It does not mean in this case that all academic scientists should promptly engage in defense research and development projects, for it is essential in the long run that our strength in teaching and fundamental research be maintained and increased. It does mean that we should have a balance, appropriate to the times, between the normal functions of academic science and the abnormal and immediate demands of military problems. (Abstracted from *Science and Mobilization*, report of the Science Advisory Committee of the Office of Defense Mobilization, OLIVER E. BUCKLEY, Chairman.)